JBF 1769
26 April 2002 Disk used
ARTICLE IN PRESS
No. of Pages 16, DTD = 4.3.1
SPS, Chennai

ELSEVIER

Journal of Banking & Finance xxx (2002) xxx–xxx

Journal of BANKING & FINANCE

www.elsevier.com/locate/econbase

Risk, returns, and values in the presence of differential taxation

Simon Benninga [a,b,*], Oded Sarig [b,c]

[a] Faculty of Management, Tel-Aviv University, Israel Wharton School, University of Pennsylvania, 69978 Tel-Aviv, Israel
[b] Wharton School, University of Pennsylvania, USA
[c] The Arison School of Business, IDC, Herzliya, Israel

Received 23 July 2001; accepted 28 January 2002

Abstract

We show that there exist separate separate security market lines (SMLs) for debt and equity securities in an equilibrium with differential taxation of debt and equity. We characterize the conditions under which these SMLs have the same price of risk (with different intercepts) and the conditions under which the tax effect of leverage is linear in debt value as in the adjusted present value method. We explore the implications of our results for cost of capital calculations: How to calculate the cost of capital for debt and equity and how to unlever betas correctly accounting for differential taxation. © 2002 Published by Elsevier Science B.V.

JEL classification: G12; G31
Keywords: CAPM; SML; Cost of capital; Differential taxation

1. Introduction

In this paper we derive risk-return relations for differentially taxed debt and equity securities. When a tax code specifies differential taxation for assets it typically restricts asset holdings as well to prevent tax arbitrage. More importantly, such restrictions are generically necessary for the existence of equilibrium. [1] Hence, equilibrium

* Corresponding author. Tel.: +972-3-640-6317; fax: +972-3-640-8518.
E-mail addresses: benninga@post.tau.ac.il (S. Benninga), sarig@idc.ac.il (O. Sarig).

[1] See, for example, Schaefer (1982) and Talmor (1989). Dammon and Green (1987) and Dammon (1988) show that restrictions on investor tax heterogeneity and on the span on the market may make an equilibrium possible without short-sale restrictions.

0378-4266/02/$ - see front matter © 2002 Published by Elsevier Science B.V.
PII: S0378-4266(02)00249-2

in a differential-taxation economy is generally characterized by market segmentation, which implies different time values of money and different prices of risk in different market segments. In the context of the CAPM, for example, this means that the separate security market line (SMLs) of these market segments have different intercepts and different slopes.

Corporate finance theory and practice tend to ignore this aspect of differentially taxed capital markets: A typical cost of capital calculation takes the time value of money and the price of risk to be (mistakenly, as we later show) the same for debt and equity. One reason for this is that the finance literature dealing with equilibrium and taxation has been almost exclusively concerned with dividend versus ordinary income taxes. [2] There is, however, very sparse literature on risk-return tradeoffs when debt and equity are differentially taxed: Theories of capital structure typically examine the valuation effects of taxation and ignore the issue of risk-return tradeoff while theories of risk-return tradeoff typically ignore the differential taxation of risky asset classes. [3] Consequently, academics and practitioners alike use a number of ad hoc approaches to determine the appropriate risk-free benchmark returns and risk adjustments for taxable cash flows.

The ad hoc nature of the formulas used to value risky taxable cash flows means that the underpinnings of these formulas are often left unspecified and no proof is offered to show that the formulas are consistent with equilibrium. Consider, for example, the formulas to convert estimated betas of stocks and bonds to asset betas (i.e., to "unlever" equity betas). Some authors (e.g., Myers and Ruback, 1992; Gilson et al., 1998) calculate asset betas simply as the weighted averages of the betas of the firm's debt and equity. Others, such as Kaplan and Ruback (1995), scale down the debt's beta as well as its weight by one minus the corporate tax rate. (Some textbooks, such as Ross et al. (1993), also use this unlevering formula.) These two different procedures yield different risk-adjusted discount rates even though both are supposed to correspond to the same setting—the Modigliani and Miller (1963) setting where interest is a deductible expense at the corporate level and there are no personal taxes. More importantly, independent of the unlevering formula used, the very idea that debt and equity betas can be unlevered to estimate asset betas *implicitly assumes that the same price of risk applies to all beta estimates* since the same market risk premium is applied to *all* beta averages.

A similar confusion is evident in the use of the risk-free benchmark returns. A few of the open issues are: Should the risk-free return for equity be before tax (as many textbooks suggest) or after tax (as Ruback (1986) suggests)? If the equity risk-free

---

[2] See, for example, Long (1977), Elton and Gruber (1978), Litzenberger and Ramaswamy (1979), Singer (1979), Constantinides (1983, 1984), Dammon and Green (1987), Ross (1987), Dammon (1988), Dammon et al. (1989), Bossaerts and Dammon (1994), Dammon and Spatt (1996).

[3] An exception is Hamada (1969, 1972) who analyzes the risk-return implications of the capital structure theory in the presence of asymmetric corporate taxation of Modigliani and Miller (1963). The literature on international taxation (e.g., Black, 1974 or Gordon and Varian, 1989) does deal with segmentation, though in a context different from the one considered here.

JBF 1769
26 April 2002 Disk used
ARTICLE IN PRESS
No. of Pages 16, DTD = 4.3.1
SPS, Chennai

return should be after tax, after *what* tax? Should the risk-free return be the same for debt and equity securities?

The disjoint analysis of tax effects and risk adjustments is in part due to the perception that there exists no integrated equilibrium model of capital asset pricing that incorporates both corporate and personal taxation. This perception is probably driven by the fact that the existence of differential tax rates on personal income necessitates some restrictions on asset holdings (typically in the form of short-sale constraints). The reason such restrictions are needed is that in their absence there may be unbounded tax arbitrage opportunities, which entail unbounded investor opportunity sets. This means that the first-order conditions for unconstrained asset allocation do not hold for all individuals and for all assets. The common perception is that, while it is not difficult to incorporate these restrictions in a model, it is well nigh impossible to manipulate the model to get a simple linear equation for the risk-return tradeoff. We address this difficulty and show that it can be overcome.

We address another, related, question: The interaction of leverage, taxation, and value in a differentially taxed world. The analysis of this issue is typically done using the adjusted present value (APV) tool. In the standard application of APV, the tax effects of leverage are taken to be some fraction of debt value. It is not clear, however, that the value of the debt is a sufficient statistic for the impact of taxation on firm value. To see how this may happen, consider two debt securities whose cash flows are *not* the same but whose market values *are* equal. If the tax implications of debt financing are *state dependent*, the overall tax impact of these two debt issues need not be the same. In other words, the market value of the debt need not be a sufficient statistic of which one can calculate the tax impact of leverage, as one does in the standard application of the APV method. In the context of our preference-free model, we find necessary and sufficient conditions for the market value of the debt to be a sufficient statistic of which the APV of a firm can be calculated, and show that these conditions are analogous to the necessary and sufficient conditions for the price of risk being the same in the debt and equity markets. This means that, when using APV one implicitly assumes similar conditions under which the use of weighted average cost of capital (WACC) (i.e., the unlevering of either equity risk or equity return) is valid—assuming equality of the price of risk across security types.

The structure of this paper is as follows: In Section 2 we derive an equilibrium model with differential taxation of equity and debt. The results of this model are applied in Section 3 to show that there are two SMLs, one for debt securities and one for equity securities. In Section 3 we also derive the necessary and sufficient conditions under which the tax effect of leverage is a function of the market value of a firm's debt. In Section 4 we discuss the application of our results to the standard CAPM. In Section 5 we apply the results to the calculation of asset betas and the WACC. Section 6 concludes.

2. The model

We derive our results within a standard two-date state-preference model with multiple consumers (also called "investors") and multiple firms similar to that of DeAngelo and Masulis (1980). There are two dates, 0 and 1. At date 0 firms issue securities and consumers invest their wealth in a portfolio of securities that payoff at date 1. We assume that there exist all possible state securities—securities that pay a unit of consumption at date 1 if a given state of nature occurs and zero otherwise. We also assume that there are no bankruptcy costs, agency costs, etc., exclusively focusing on the risk-return tradeoff in the presence of differential taxation.

State securities can be either *debt* or *equity*. The type of the security determines its taxation both at the investor level and at the corporate level. The payoff of a debt security that pays off in state s is taxed when received by investor i at the rate $\tau^i_{\mathrm{PD}}(s)$. Hence, purchasing a unit of this security for price $P_{\mathrm{D}}(s)$ at date 0 will give consumer i an after-tax payoff of $(1-\tau^i_{\mathrm{PD}}(s))$ in state s at date 1. [4] Similarly, the purchase of one unit of an equity security today at price $P_{\mathrm{E}}(s)$ results in an after-tax cash flow to the consumer of $(1-\tau^i_{\mathrm{PE}}(s))$ in state s at date 1. We assume that, state by state, personal tax rates on debt income are higher than the personal tax rates on equity income: $1 > \tau^i_{\mathrm{PD}}(s) > \tau^i_{\mathrm{PE}}(s) \geqslant 0\ \forall i, s$. We assume that there exists a fixed, strictly positive, supply of the full set of both equity and debt state securities. This means that financial markets are *doubly complete*.

Under double completeness, investors may attempt to tax arbitrage across the debt and equity markets; hence investor opportunity sets may be unbounded and no equilibrium may exist. Tax codes, however, prohibit unbounded tax arbitrage by restricting the tax deductibility of asset returns, which effectively restrict investors' asset holdings. To capture these restrictions in our model, we assume that short sales are not allowed. [5] Note that, since the debt and the equity markets are both complete, the investment opportunity sets of investors are not restricted by the short-sale constraints.

Firms are endowed with state-dependent, before-interest, before-corporate-tax income, which is assumed to reflect each firm's optimal investment decisions. Firms are taxed at the rate $\tau_{\mathrm{C}}(s)$ in state s when their income is positive. Since we are dealing with a single-period model, there is no carry backward or forward of losses; when corporate income is negative the corporation's tax rate is zero. Payments to debt holders are tax deductible expenses while payments to equity holders are not.

3. Equilibrium asset prices

Investor i chooses her holdings $x^i_{\mathrm{E}}(s)$ and $x^i_{\mathrm{D}}(s)$ in state-s equity securities and state-s debt securities, respectively, to solve the following utility maximization problem:

[4] We assume that the whole payoff is taxed. This is the appropriate equivalent in a one-period framework to taxation of rates of returns only in an infinite-horizon framework.

[5] This assumption can be weakened somewhat (see Dammon and Green, 1987; Talmor, 1989).

$$
\begin{aligned}
\underset{x^i_{\mathrm{E}}(s),x^i_{\mathrm{D}}(s)}{\mathrm{Max}}\; u_i(c_{i0}) + \delta_i E[u_i(\tilde{c})] &= u_i(c_{i0}) + \delta_i \sum_s \pi(s) u_i(c_{is}) \\
&= u_i(c_{i0}) + \delta_i \sum_s \pi(s) u_i\big[x^i_{\mathrm{E}}(s)\big(1-\tau^i_{\mathrm{PE}}(s)\big) \\
&\quad + x^i_{\mathrm{D}}(s)\big(1-\tau^i_{\mathrm{PD}}(s)\big)\big] \quad \text{s.t. } x^i_{\mathrm{E}}(s),\; x^i_{\mathrm{D}}(0) \geqslant 0, \\
c_{i0} &+ \sum_s x^i_{\mathrm{E}}(s)P_{\mathrm{E}}(s) + x^i_{\mathrm{D}}(s)P_{\mathrm{D}}(s) = W_i
\end{aligned} \tag{1}
$$

c_{i0} is consumer i's date-0 consumption, $\tilde{c} = \{c_{i1}, \ldots, c_{is}\}$ is her date-1 consumption in each state, δ_i is her rate of time preference, and W_i is her wealth. The state probabilities, $\pi(s)$, need not be homogeneous.

It can be shown using standard tools of general equilibrium analysis (e.g., Debreu, 1959) that an equilibrium with positive prices for equity and debt state securities exists if some mild conditions are assumed about investor beliefs and preferences.

3.1. Consumer indifference between purchasing debt and equity

Investor i will be indifferent between purchasing a state-s *equity* security and purchasing a state-s *debt* security if and only if a given expenditure at date 0 yields the same state-s consumption:

$$x_{\mathrm{E}}P_{\mathrm{E}}(s) = x_{\mathrm{D}}P_{\mathrm{D}}(s) \iff (1-\tau^i_{\mathrm{PD}}(s))x_{\mathrm{D}} = (1-\tau^i_{\mathrm{PE}}(s))x_{\mathrm{E}} \tag{2}$$

Relation (2) means that consumer i will be indifferent between state-s equity and debt if and only if:

$$\frac{(1-\tau^i_{\mathrm{PD}}(s))}{P_{\mathrm{D}}(s)} = \frac{(1-\tau^i_{\mathrm{PE}}(s))}{P_{\mathrm{E}}(s)} \tag{3}$$

We assume that for each state s there exists a consumer $i(s)$ who is indifferent between purchasing state-s debt and equity securities. A sufficiently rich range of relative tax rates will imply satisfaction of this assumption. We denote the state-s break-even ratio of personal tax rates by $\varphi(s)$:

$$\varphi(s) \equiv \frac{(1-\tau^{i(s)}_{\mathrm{PD}}(s))}{(1-\tau^{i(s)}_{\mathrm{PE}}(s))} = \frac{P_{\mathrm{D}}(s)}{P_{\mathrm{E}}(s)} \tag{4}$$

Since investors are, by assumption, taxed less heavily on debt income than on equity income, $0 < \varphi(s) < 1$ and hence $P_{\mathrm{D}}(s) = \varphi(s)P_{\mathrm{E}}(s) < P_{\mathrm{E}}(s)$.

3.2. Security pricing in equilibrium

Consider an arbitrary equity security that pays a random before-personal-tax payoff of $\widetilde{Q}_{\mathrm{E}} = \{Q_{\mathrm{E}}(s)\}$. The time-0 price of this security is $V_{\mathrm{E}} = \sum_s Q_{\mathrm{E}}(s)P_{\mathrm{E}}(s)$. Similarly, the price of a debt security with time-1 payoffs $\widetilde{Q}_{\mathrm{D}} = \{Q_{\mathrm{D}}(s)\}$ is $V_{\mathrm{D}} = \sum_s Q_{\mathrm{D}}(s)P_{\mathrm{D}}(s) = \sum_s Q_{\mathrm{D}}(s)\varphi(s)P_{\mathrm{E}}(s)$.

When the tax system differentiates between debt income and equity income, it is necessary to distinguish between the risk-free rate of *debt* securities and the risk-free rate of *equity* securities:

$$\mathrm{Rf}_{\mathrm{E}} \equiv 1 + \mathrm{rf}_{\mathrm{E}} = 1 / \sum_s P_{\mathrm{E}}(s)$$

and

$$\mathrm{Rf}_{\mathrm{D}} \equiv 1 + \mathrm{rf}_{\mathrm{D}} = 1 / \sum_s P_{\mathrm{D}}(s)$$

Since equity income is less heavily taxed than debt income, $P_{\mathrm{D}}(s) < P_{\mathrm{E}}(s)\ \forall s$ so that the risk-free return for equity securities is lower than the risk-free rate for debt securities:

$$\mathrm{Rf}_{\mathrm{D}} = \frac{1}{\sum_s P_{\mathrm{D}}(s)} = \frac{1}{\sum_s \varphi(s) P_{\mathrm{E}}(s)} > \frac{1}{\sum_s P_{\mathrm{E}}(s)} = \mathrm{Rf}_{\mathrm{E}} \tag{5}$$

Next we consider the expected returns of *risky* securities. (All proofs are in Appendix A.)

Proposition 1. *There are separate SML for debt and equity securities*:

$$E(\widetilde{R}_{\mathrm{E}}) = \mathrm{Rf}_{\mathrm{E}} - \mathrm{Cov}\left(\frac{\mathrm{Rf}_{\mathrm{E}}\widetilde{P}_{\mathrm{E}}}{\tilde{\pi}}, \widetilde{R}_{\mathrm{E}}\right) \equiv \mathrm{Rf}_{\mathrm{E}} - \mathrm{Cov}(\widetilde{M}_{\mathrm{E}}, \widetilde{R}_{\mathrm{E}}) \tag{6}$$

$$E(\widetilde{R}_{\mathrm{D}}) = \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}\left(\frac{\widetilde{P}_{\mathrm{D}}}{\tilde{\pi}}, \widetilde{R}_{\mathrm{D}}\right)\mathrm{Rf}_{\mathrm{D}} \equiv \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}(\widetilde{M}_{\mathrm{D}}, \widetilde{R}_{\mathrm{D}}) \tag{7}$$

where $\widetilde{M}_{\mathrm{E}}$ *and* $\widetilde{M}_{\mathrm{D}}$ *are the equity and debt pricing kernels.* [6]

Eqs. (6) and (7) imply that, in general, the SMLs for debt securities and for equity securities have different intercepts and different slopes. This means that the expected rates of return of equity and debt securities with identically distributed *before-personal-tax* payoffs are, in general, different. The different pricing of debt and equity securities is due to the differential taxation of debt and equity and is sustained by the restrictions on tax arbitrage, which entail segmentation between investors who hold state-*s* debt and equity securities.

Proposition 2. *The price of risk is the same for debt and equity securities* (*i.e.*, *the equity SML and the debt SML are parallel*, *albeit with different intercepts*) *if and only if for every state s* $\varphi(s) = \varphi$.

In Section 4, we use the following corollary to Proposition 2:

[6] In Section 4 we rederive these SMLs in their more familiar form by assuming that there are traded assets that are perfectly correlated with the pricing kernels.

JBF 1769
26 April 2002 Disk used
ARTICLE IN PRESS
No. of Pages 16, DTD = 4.3.1
SPS, Chennai

Corollary. *The pricing kernel for debt and equity securities is the same if and only if* $\varphi(s) = \varphi$ *for all s.*

Proposition 2 shows that the necessary and sufficient condition for the SMLs of debt and equity securities to be parallel is that *state by state* the relative pricing of debt income and equity income is the same, i.e., that the marginal relative tax rate, $\varphi(s)$, is state independent. While the marginal investor need not be the same in each state, the marginal tax rates on equity and debt incomes should have a constant *ratio* in *all* states. (Below, we show that this happens in an extension of Miller (1977) equilibrium.)

Proposition 2 also provides a necessary and sufficient condition to answer one of the questions posed in the introduction: *When the break-even ratio of tax rates is state-independent, the price of risk is the same for debt and equity securities.* Intuitively, when the relative pricing of debt and equity is state-independent, the covariance terms in (6) and (7) capture the relative taxation of debt and equity incomes.

As we show next, the condition of Proposition 2—i.e., that the break-even tax rate $\varphi(s)$ is state independent—is also related to the standard APV method for the valuation of levered firms. Suppose we write—as in the standard APV method to valuing leverage—$\Delta V \equiv V_L - V_U = KV_D$, where V_L and V_U are, respectively, the values of the same firm with and without leverage, V_D is the market value of the levered firm's debt, and K is some constant that reflects the tax-related valuation impact of leverage. Implicit in this adjustment is the assumption that debt value, V_D, is a sufficient statistic for the valuation impact of leverage; otherwise K should depend on the particular distribution of the debt payments.

Proposition 3. *The adjustment of the value of the firm for the tax effect of leverage is independent of the shape of the distribution of the debt payments* (*i.e., the market value of the debt is a sufficient statistic to compute the tax effect of leverage*) *if and only if* $(P_E(s)[1 - \tau_C(s)])/(P_D(s))$ *is state-independent. Since* $P_E(s)/P_D(s) = 1/\varphi(s)$, *this condition is equivalent to the state-independence of* $([1 - \tau_C(s)])/\varphi(s)$.

If, as is usually assumed, the statutory corporate tax rate is state-independent (i.e., $\tau_C(s) = \tau_C$), then the standard value additivity approach to valuing leverage holds if and only if the relative personal taxation of debt and equity income $\varphi(s)$ is state-independent. By Proposition 2 this condition is also necessary and sufficient for a single price of risk for debt and equity returns (i.e., for parallel debt and equity SMLs). Proposition 3, therefore, means that the two standard tools of corporate finance:

- Adjusting the risk-free returns while keeping the price of risk equal across security classes, and
- Taking the effect of taxation to be independent of the payoff pattern of the firm's debt in APV share a common underlying implicit assumption—that the relative taxation of debt and equity income at the personal level are state-independent.

Note that Proposition 3 identifies the constant, K, in the APV method. When the corporate tax rate is state-independent and the conditions of Proposition 2 hold—$\tau_C(s) = \tau$, $\varphi(s) = \varphi$, then:

$$\Delta V \equiv V_L - V_U = \left(1 - \frac{1-\tau_C}{\varphi}\right) V_D = K V_D$$

3.3. The price of risk and APV in an extended version of Miller (1977)

In general, the marginal relative tax rates of debt and equity incomes need not be the same in all states. Consequently, Propositions 2 and 3 imply that the standard tools of corporate finance—unlevering betas and APV—cannot be applied. Yet, because corporations adjust the amounts of debt and equity securities they issue, a state-by-state extension of Miller (1977) shows that $\varphi(s) = 1 - \tau_C(s)$ in all states. We call this the extended Miller equilibrium (EME).

Proposition 4. *In an EME if the statutory corporate tax rate is state independent* (*i.e.*, $\tau_C(s) = \tau_C\ \forall s$), *the debt and equity SMLs are parallel and the risk-free benchmark return for equity securities is the after-corporate-tax risk-free benchmark return for debt securities.*

Proposition 4 answers another question posed in the introduction: What are the relative risk-free benchmark returns of equity and debt securities? In an EME when the statutory corporate tax rate is state-independent, the risk-free benchmark return for *equity* securities is the risk-free benchmark return for *debt* securities *after corporate tax*. In this case, the slopes of the equity and debt SMLs are equal.

4. Debt and equity SMLs with differential taxation

Next, we convert the state-preference setting into a standard CAPM setting where the pricing kernel is identified with a traded asset and the risk-return tradeoffs for debt and equity are estimable.

Assume that there exists a portfolio the return of which, $\widetilde{R}_{mE}$, is perfectly correlated with the equity pricing kernel: $\widetilde{M}_E = a + b\widetilde{R}_{mE}$. We call this portfolio "the equity market portfolio". Using the equity market portfolio we can derive a standard CAPM:

$$E(\widetilde{R}_E) = \mathrm{Rf}_E - \mathrm{Cov}(\widetilde{M}_E, \widetilde{R}_E) = \mathrm{Rf}_E - \mathrm{Cov}(a + b\widetilde{R}_m, \widetilde{R}_E)$$
$$= \mathrm{Rf}_E - b\,\mathrm{Cov}(\widetilde{R}_{mE}, \widetilde{R}_E) \tag{8}$$

Since, in particular, this relation is true for the equity market portfolio itself, it follows that

$$E(\widetilde{R}_{mE}) = \mathrm{Rf}_E - b\,\mathrm{Cov}(\widetilde{R}_{mE}, \widetilde{R}_{mE}) \Rightarrow -b = \frac{E(\widetilde{R}_{mE}) - \mathrm{Rf}_E}{\sigma^2_{mE}} \tag{9}$$

where σ^2_{mE} is the variance of the return of the equity market portfolio. Substituting into the expression for the pricing of equity securities gives the standard equity CAPM:

$$E(\widetilde{R}_{\mathrm{E}}) = \mathrm{Rf}_{\mathrm{E}} + \frac{\mathrm{Cov}(\widetilde{R}_{\mathrm{mE}}, \widetilde{R}_{\mathrm{E}})}{\sigma^2_{\mathrm{mE}}}[E(\widetilde{R}_{\mathrm{mE}}) - \mathrm{Rf}_{\mathrm{E}}] \equiv \mathrm{Rf}_{\mathrm{E}} + \beta_{\mathrm{E}}\Pi_{\mathrm{mE}} \tag{10}$$

where Π_{mE} is the risk premium on the equity market portfolio and β_{E} is the beta coefficient of the return of an *equity* security vis-à-vis the return of the *equity* market portfolio.

Assuming that there is also a portfolio the returns of which are perfectly correlated with the debt-market pricing kernel, we get a similar relation for the debt market:

$$\begin{aligned} E(\widetilde{R}_{\mathrm{D}}) &= \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}(\widetilde{M}_{\mathrm{D}}, \widetilde{R}_{\mathrm{D}}) = \mathrm{Rf}_{\mathrm{D}} + \frac{\mathrm{Cov}(\widetilde{R}_{\mathrm{mD}}, \widetilde{R}_{\mathrm{D}})}{\sigma^2_{\mathrm{mD}}}\Pi_{\mathrm{mD}} \\ &\equiv \mathrm{Rf}_{\mathrm{D}} + \beta_{\mathrm{D}}\Pi_{\mathrm{mD}} \end{aligned} \tag{11}$$

where Π_{mD} is the risk premium on the debt market portfolio and β_{D} is the beta coefficient of the return of a *debt* security vis-à-vis the *debt* market portfolio.

Recall that the SMLs of the two markets will, in general, have different intercepts and different slopes. Using Propositions 2, 4, and 4, we can relate the pricing parameters of the two SML equations:

Proposition 5. *In an EME in which the corporate tax rate is state independent, the SMLs of equity and debt securities are*: **The equity SML:** $E(\widetilde{R}_{\mathrm{E}}) = \mathrm{Rf}_{\mathrm{D}}(1 - \tau_{\mathrm{C}}) + \beta_{\mathrm{D}}\Pi_{\mathrm{mE}}$ **The debt SML:** $E(\widetilde{R}_{\mathrm{D}}) = \mathrm{Rf}_{\mathrm{D}} + \beta_{\mathrm{D}}\Pi_{\mathrm{mE}}$*Note that risk premiums are measured relative to the equity market portfolio in both markets*, $\Pi_{\mathrm{mE}} = E(\widetilde{R}_{\mathrm{mE}}) - \mathrm{Rf}_{\mathrm{E}}(1 - \tau_{\mathrm{C}})$, *and that both betas are estimated vis-a-vis the equity market portfolio*.

Proposition 4 characterizes the relation between the risk-free returns of equity and debt when the EME holds: $\mathrm{Rf}_{\mathrm{E}} = (1 - \tau_{\mathrm{C}})\mathrm{Rf}_{\mathrm{D}}$. Proposition 5 further shows that, under the same conditions, the slopes of the equity and the debt SMLs are *both* $\Pi_{\mathrm{mE}} \equiv E(\widetilde{R}_{\mathrm{Me}}) - \mathrm{Rf}_{\mathrm{D}}(1 - \tau_{\mathrm{C}})$. [7]

5. Calculating the weighted average cost of capital

One of the basic tools of corporate finance is the unlevering of the rates of returns of debt and equity to compute the WACC. The WACC is used to estimate the cost of

[7] Hamada and Scholes (1985) suggest an after-tax equity SML which is similar to that of Proposition 5; Taggart (1991) also presents a similar equity SML. Neither of these papers has the capital markets equilibrium characterization which underlies our results, nor do these papers examine the case of risky debt. Sick (1990) obtains results similar to Proposition 4, although not in a general equilibrium framework.

capital with which projects are evaluated. An equivalent computation is to unlever the betas of debt and equity and use an "asset" beta and an "asset" SML to estimate the cost of capital. Obviously, an "asset" SML exists only if the SMLs of debt and equity securities are parallel: If the price of risk is different for debt and equity no single price of risk applies to all "unlevered" betas.

In the preceding sections, we derive the necessary and sufficient for the price of risk to be the same in the debt and equity markets and, as a special case, the SML equations when corporations may adjust the mix of debt and equity securities they issue, i.e., in an EME. These SML equations allow us to derive the correct way to unlever equity and debt betas and estimate the cost of capital using the unlevered beta and the asset SML:

Proposition 6. *Consider a firm financed with a proportion* $x_E = E/(E+D)$ *of equity and* $x_D = 1 - x_E$ *debt, where E and D denote the values of the equity and debt, respectively. In an EME with a state-independent statutory corporate tax rate, the equity and debt betas are unlevered to an asset beta by*:

$$\beta_{\text{Assets}} = x_E \beta_{\text{Equity}} + x_D \beta_{\text{Debt}} (1 - \tau_C)$$

Furthermore, the asset SML used in calculating the required return on the assets relative to asset betas is the equity SML:

$$E(\widetilde{R}_{\text{Assets}}) = \text{Rf}_D (1 - \tau_C) + \beta_{\text{Assets}} \Pi_{mE}$$

Thus, the risk-free benchmark return to determine expected returns on assets is the same as the equity risk-free benchmark—the after-*corporate*-tax risk-free return on debt securities. Note that the debt and equity betas are both estimated relative to the *equity* market portfolio. Hence, the estimated *debt* beta is "inflated" compared to the beta that would have been estimated against the return of the *debt* market portfolio. Accordingly, when averaged with the equity beta to estimate the asset beta, the inflated debt beta is deflated back by $(1 - \tau_C)$.

In an EME, the risk-free rate for free cash flows is the one suggested by Ruback (1986). Proposition 6 generalizes his result for all patterns of asset, debt, and equity cash flows, and derives the correct unlevering equation for betas. It further shows that this risk-free rate and unlevering relation are conditional on the existence of an EME.

6. Concluding remarks

We analyze the risk-return tradeoff for stocks and bonds when their income streams are differentially taxed. We characterize the risk-free returns, risk measures, and risk prices in the debt and equity market segments.

When assets are differentially taxed there are typically constraints on investor asset holdings that inhibit tax arbitrage across asset classes. Such restrictions induce market segmentation, which generically leads to differential pricing of risk-free and risky cash flows. Moreover, asset-holding constraints mean that the first-order conditions of unconstrained asset demands no longer describe the risk-return tradeoff since non-zero shadow prices are typically associated with these constraints. Nonetheless, we derive necessary and sufficient conditions under which the price of risk in the debt and equity market segments is the same, i.e., the debt and equity SMLs are parallel. We also show that the conditions under which the SMLs are parallel are closely related to the conditions under which the standard APV method to valuing leverage holds: Under these conditions, it is appropriate to compute the tax effect of leverage based on debt value alone.

The conditions necessary for equality of the price of risk in the debt and equity markets and for the application of APV obtain when corporations adjust their security offerings in an equilibrium that extends Miller (1977) to an uncertain world—EME. We show that, in an EME, the risk-free return for equity securities is the after-corporate tax yield to maturity of a risk-free debt. We also show that, in an EME, the risk premiums of *both* equity and debt securities are the product of the asset betas measured relative to the *equity* market index and the risk premium of this index.

Finally, we consider the cost of capital with which investments are evaluated. We show that, in an EME, the asset cost of capital is computed-off the *equity* SML. We also show that to convert equity and debt betas to an asset beta (i.e., to "unlever" equity beta) one needs to use a weighted average beta estimate in which debt beta is deflated by one minus the corporate tax rate.

7. Uncited references

Aivazian and Callen, 1987, Bosshardt, 1984, Dybvig and Ross, 1986.

Acknowledgements

We thank an anonymous referee, Yakov Amihud, Shmuel Kandel, Roni Michaely, Avi Wohl, and seminar participants at the University of Groningen and Tel-Aviv University for helpful comments and suggestions. Benninga acknowledges a grant from the Israel Institute of Business Research at Tel-Aviv University.

Appendix A. Proofs of propositions

Proof for Proposition 1. Using a technique first introduced by Beja (1972), we rewrite the price of a risky equity security that has a before-personal-tax payoff vector $\widetilde{Q}_{\mathrm{E}}$ as:

$$V_{\mathrm{E}} = \sum_s P_{\mathrm{E}}(s)Q_{\mathrm{E}}(s) = \sum_s \pi(s)\frac{P_{\mathrm{E}}(s)}{\pi(s)}Q_{\mathrm{E}}(s) = E\left[\left(\frac{P_{\mathrm{E}}(s)}{\pi(s)}\right)Q_{\mathrm{E}}(s)\right]$$
$$= E\left(\frac{P_{\mathrm{E}}(s)}{\pi(s)}\right)E(\widetilde{Q}_{\mathrm{E}}) + \mathrm{Cov}\left(\frac{\widetilde{P}_{\mathrm{E}}}{\tilde{\pi}}, \widetilde{Q}_{\mathrm{E}}\right) = \frac{1}{\mathrm{Rf}_{\mathrm{E}}}E(\widetilde{Q}_{\mathrm{E}}) + \mathrm{Cov}\left(\frac{\widetilde{P}_{\mathrm{E}}}{\tilde{\pi}}, \widetilde{Q}_{\mathrm{E}}\right)$$

Bringing Rf_{E} inside the parentheses and using $\widetilde{R}_{\mathrm{E}} = \widetilde{Q}_{\mathrm{E}}/V_{\mathrm{E}}$ to get the equity pricing kernel, $\widetilde{M}_{\mathrm{E}} \equiv (\mathrm{Rf}_{\mathrm{E}}\widetilde{P}_{\mathrm{E}})/\tilde{\pi}$, gives the equity market SML. □

Proof for Proposition 2. *Sufficiency*: Assume that $\varphi(s) = \varphi$ for every state s. Then

$$E(\widetilde{R}_{\mathrm{D}}) = \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}\left(\frac{\widetilde{P}_{\mathrm{D}}}{\tilde{\pi}}, \widetilde{R}_{\mathrm{D}}\right)\mathrm{Rf}_{\mathrm{D}} = \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}\left(\frac{\varphi\widetilde{P}_{\mathrm{D}}}{\tilde{\pi}}, \widetilde{R}_{\mathrm{D}}\right)\frac{\mathrm{Rf}_{\mathrm{D}}}{\varphi}$$
$$= \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}\left(\frac{\widetilde{P}_{\mathrm{E}}}{\tilde{\pi}}, \widetilde{R}_{\mathrm{D}}\right)\mathrm{Rf}_{\mathrm{E}} = \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}\left(\frac{\widetilde{P}_{\mathrm{E}}\mathrm{Rf}_{\mathrm{E}}}{\tilde{\pi}}, \widetilde{R}_{\mathrm{D}}\right)$$
$$= \mathrm{Rf}_{\mathrm{D}} - \mathrm{Cov}(\widetilde{M}_{\mathrm{E}}, \widetilde{R}_{\mathrm{D}})$$

Necessity: Assume that the SMLs are parallel (with possibly different intercepts). This means that the risk premium of any debt security can be determined by using either the debt or the equity kernel:

$$E(\widetilde{R}_{\mathrm{D}}) - \mathrm{Rf}_{\mathrm{D}} = \mathrm{Cov}(\widetilde{M}_{\mathrm{E}}, \widetilde{R}_{\mathrm{D}}) = \mathrm{Cov}(\widetilde{M}_{\mathrm{D}}, \widetilde{R}_{\mathrm{E}})$$

In particular, this is true for a state-s debt security. Let $1_{\mathrm{D}}(s)$ be a debt security that pays off \$1 in state s only. For this security the covariance of its payoff with the *equity* market kernel is

$$\mathrm{Cov}(\widetilde{M}_{\mathrm{E}}, 1_{\mathrm{D}}(s)) = \mathrm{Cov}\left(\frac{P_{\mathrm{E}}(s)\mathrm{Rf}_{\mathrm{E}}}{\pi(s)}, 1_{\mathrm{D}}(s)\right) = P_{\mathrm{E}}(s)\mathrm{Rf}_{\mathrm{E}} - \pi(s)$$

Similarly, the covariance of this security's payoff with the *debt* market kernel is:

$$\mathrm{Cov}(\widetilde{M}_{\mathrm{D}}, 1_{\mathrm{D}}(s)) = P_{\mathrm{D}}(s)\mathrm{Rf}_{\mathrm{D}} - \pi(s)$$

Equating these expressions shows that $\varphi(s) = (P_{\mathrm{D}}(s)/P_{\mathrm{E}}(s)) = (\mathrm{Rf}_{\mathrm{E}}/\mathrm{Rf}_{\mathrm{D}})$ for all s.

Corollary. *The equity and debt pricing kernels are* $M_{\mathrm{E}}(s) = (P_{\mathrm{E}}(s)\mathrm{Rf}_{\mathrm{E}})/\pi(s)$, $M_{\mathrm{D}}(s) = (P_{\mathrm{D}}(s)\mathrm{Rf}_{\mathrm{D}})/\pi(s)$. *Since under the conditions of the corollary* $P_{\mathrm{D}}(s) = \varphi P_{\mathrm{E}}(s)$, *the result follows immediately from the definitions of the risk-free rates.* □

Proof for Proposition 3. Let the firm's after-corporate-tax cash flows in state s be denoted by $\mathrm{FCF}(s)$ and the state-s payments to the debt holders be denoted by $\mathrm{CF}_{\mathrm{D}}(s)$. Then, the value of the unlevered firm is

$$V_{\mathrm{U}} = \sum_s P_{\mathrm{E}}(s)\mathrm{FCF}(s)$$

and the value of the levered firm is:

$$V_L = V_E + V_D = \sum_s P_E(s)\{\mathrm{FCF}(s) - \mathrm{CF}_D(s)[1 - \tau_C(s)]\} + \sum_s P_D(s)\mathrm{CF}_D(s)$$

A simple manipulation gives:

$$\Delta V \equiv V^L - V^U = \sum_s \mathrm{CF}_D\{P_D(s) - P_E(s)[1 - \tau_C(s)]\}$$
$$= \sum_s \mathrm{CF}_D P_D(s)\left\{1 - \frac{P_E(s)[1-\tau_C(s)]}{P_D(s)}\right\}$$

Sufficiency: If $1 - ((P_E(s)[1-\tau_C(s)])/P_D(s)) = K\ \forall s$, then above equation simplifies to:

$$\Delta V = \sum_s \mathrm{CF}_D P_D(s)\left\{1 - \frac{P_E(s)[1-\tau_C(s)]}{P_D(s)}\right\} = \sum_s \mathrm{CF}_D P_D(s) K = K V_D$$

Necessity: If $\Delta V = KV_D$ for all debt payoff patterns, then it is also true for debt state securities, $1_D(s)$. The impact of issuing this security on the value of the firm is:

$$\Delta V = \sum_s 1_D(s) P_D(s)\left\{1 - \frac{P_E(s)[1-\tau_C(s)]}{P_D(s)}\right\} = P_D(s) - P_E(s)[1-\tau_C(s)]$$

By assumption $\Delta V = KV_D$. Since the value of a state-*s* debt security is $P_D(s)$, we get:

$$\Delta V = P_D(s) - P_E(s)[1-\tau_C(s)] = KP_D(s) \Rightarrow \left\{1 - \frac{P_E(s)[1-\tau_C(s)]}{P_D(s)}\right\}$$
$$= K \quad \forall s \qquad \square$$

Proof for Proposition 4. The equality of the price of risk result directly from Proposition 2 and the corollary to Proposition 4. The relation between the risk-free returns follows from:

$$\mathrm{Rf}_D = \frac{1}{\sum_s P_D(s)} = \frac{1}{\sum_s \varphi(s) P_E(s)} = \frac{1}{\varphi \sum_s P_E(s)} = \frac{1}{(1-\tau_C)\sum_s P_E(s)}$$
$$= \frac{1}{(1-\tau_C)}\mathrm{Rf}_E \qquad \square$$

Proof for Proposition 5. When corporate tax is state-independent, $\widetilde{R}_{mD} = \widetilde{R}_{mE}/(1-\tau_C)$. Thus,

$$\frac{\mathrm{Cov}(\widetilde{R}_{mD}, \widetilde{R}_D)}{\sigma^2_{mD}}[E(\widetilde{R}_{mD}) - \mathrm{Rf}_D] = \frac{\mathrm{Cov}(\widetilde{R}_{mE}/(1-\tau_C), \widetilde{R}_D)}{\sigma^2_{mD}}[E(\widetilde{R}_{mE})/(1-\tau_C)$$
$$- \mathrm{Rf}_E/(1-\tau_C)] = \frac{\mathrm{Cov}(\widetilde{R}_{mE}, \widetilde{R}_D)}{\sigma^2_{mD}}$$
$$\times [E(\widetilde{R}_{mE}) - \mathrm{Rf}_E]$$

Since $\mathrm{Rf}_E = \mathrm{Rf}_D(1-\tau_C)$, this established the result. $\square$

Proof for Proposition 6. Let $\mathrm{CF_A}(s)$ denote state-s cash flow before debt payments and before corporate taxes. Hence, $\mathrm{FCF}(s) = \mathrm{CF_A}(s)(1-\tau_\mathrm{C})$. Let $\mathrm{CF_D}(s)$ denote state-s debt cash flows. The value of equity and debt is given by:

$$V_\mathrm{D} = \sum_s P_\mathrm{D}(s)\mathrm{CF_D}(s) = \sum_s (1-\tau_\mathrm{C})P_\mathrm{E}(s)\mathrm{CF_D}(s)$$

$$V_\mathrm{E} = \sum_s P_\mathrm{E}(s)\mathrm{CF_E}(s) = \sum_s P_\mathrm{E}(s)(\mathrm{FCF}(s) - (1-\tau_\mathrm{C})\mathrm{CF_D}(s))$$

A simple summation shows that in an EME the value of the firm is invariant to leverage:

$$V_F = V_\mathrm{D} + V_\mathrm{E} = \sum_s P_\mathrm{E}(s)\mathrm{FCF}(s)$$

This implies that firm value can be derived assuming the firm is all equity financed—from free cash flows valued by the *equity* state prices:

$$E(r_\mathrm{A}) = \mathrm{Rf_E} + \beta_\mathrm{Assets}\Pi_\mathrm{ME} \quad \text{and} \quad \beta_\mathrm{Assets} = \frac{\mathrm{Cov}[(\mathrm{FCF}/(V_\mathrm{E}+V_\mathrm{D})), R_\mathrm{ME}]}{\mathrm{Var}(R_\mathrm{ME})}$$

Next estimate β_Assets by averaging β_Debt and β_Equity. Compute the betas of the debt and the equity:

$$\beta_\mathrm{Debt} = \frac{\mathrm{Cov}((\mathrm{CF_D}/V_\mathrm{D}), \widetilde{R}_\mathrm{ME})}{\mathrm{Var}(\widetilde{R}_\mathrm{ME})} = \frac{1}{V_\mathrm{D}}\frac{\mathrm{Cov}(\mathrm{CF_D}, \widetilde{R}_\mathrm{ME})}{\mathrm{Var}(\widetilde{R}_\mathrm{ME})}$$

$$\beta_\mathrm{Equity} = \frac{\mathrm{Cov}((\mathrm{CF_E}/V_\mathrm{E}), \widetilde{R}_\mathrm{ME})}{\mathrm{Var}(\widetilde{R}_\mathrm{ME})} = \frac{1}{V_\mathrm{E}}\frac{\mathrm{Cov}(\mathrm{FCF} - \mathrm{CF_D}(1-\tau_\mathrm{C}), \widetilde{R}_\mathrm{ME})}{\mathrm{Var}(\widetilde{R}_\mathrm{ME})}$$

We want to compute β_Assets by averaging the debt and the equity βs, using market values in the weights:

$$\frac{V_\mathrm{E}}{V_\mathrm{D}+V_\mathrm{E}}\beta_\mathrm{Equity} + \frac{V_\mathrm{D}}{V_\mathrm{D}+V_\mathrm{E}}K\beta_\mathrm{Debt} = \left(\frac{1}{V_\mathrm{D}+V_\mathrm{E}}\right)\left[\frac{\mathrm{Cov}(\mathrm{FCF} - \mathrm{CF_D}(1-\tau_\mathrm{C}), \widetilde{R}_\mathrm{ME})}{\mathrm{Var}(\widetilde{R}_\mathrm{ME})}\right] + K\left(\frac{1}{V_\mathrm{D}+V_\mathrm{E}}\right)\left[\frac{\mathrm{Cov}(\mathrm{CF_D}, \widetilde{R}_\mathrm{ME})}{\mathrm{Var}(\widetilde{R}_\mathrm{ME})}\right]$$

This should equal $\beta_\mathrm{Assets} = \mathrm{Cov}[(\mathrm{FCF}/(V_\mathrm{E}+V_\mathrm{D})), R_\mathrm{ME}]/\mathrm{Var}(R_\mathrm{ME})$, which is true for all beta estimates if and only if $K = (1-\tau_\mathrm{C})$:

$$\beta_\mathrm{Assets} = \left(\frac{V_\mathrm{E}}{V_\mathrm{D}+V_\mathrm{E}}\right)\beta_\mathrm{Equity} + \left(\frac{V_\mathrm{D}}{V_\mathrm{D}+V_\mathrm{E}}\right)\beta_\mathrm{Debt}(1-\tau_\mathrm{C})$$

Accordingly, the expected return on the assets is given by:

$$E(\widetilde{R}_\mathrm{A}) = \mathrm{Rf_E} + \frac{\mathrm{Cov}(\widetilde{R}_\mathrm{A}, \widetilde{R}_\mathrm{ME})}{\mathrm{Var}(\widetilde{R}_\mathrm{ME})}\Pi_\mathrm{ME} = \mathrm{Rf_D}(1-\tau_\mathrm{C}) + \beta_\mathrm{Assets}\Pi_\mathrm{ME} \qquad \square$$

References

Aivazian, V.A., Callen, J.L., 1987. Miller's irrelevance mechanism: A note. Journal of Finance 42, 169–180.

Beja, A., 1972. On systematic and unsystematic components of financial risk. Journal of Finance 27, 37–45.

Black, F., 1974. International capital market equilibrium with investment barriers. Journal of Financial Economics 1, 337–352.

Bossaerts, P., Dammon, R.M., 1994. Tax-induce intertemporal restrictions on security returns. Journal of Finance 44, 1347–1371.

Bosshardt, D., 1984. Equity clienteles, short-sale restrictions, and the miller equilibrium. Financial Review 19–2, 242–250.

Constantinides, G.M., 1983. Capital market equilibrium with personal tax. Econometrica 51, 611–636.

Constantinides, G.M., 1984. Optimal stock trading with personal taxes. Journal of Financial Economics 13, 65–89.

Dammon, R.M., 1988. A security market and capital structure equilibrium under uncertainty with progressive income taxes. Research in Finance 7, 53–74.

Dammon, R.M., Dunn, K.B., Spatt, C.S., 1989. A reexamination of the value of tax options. Review of Financial Studies 2 (3), 341–372.

Dammon, R.M., Green, R.C., 1987. Tax arbitrage and the existence of equilibrium prices for financial assets. Journal of Finance 42, 1143–1166.

Dammon, R.M., Spatt, C.S., 1996. The optimal trading and pricing of asymmetric capital gains taxes and transaction costs. Review of Financial Studies 9, 921–952.

DeAngelo, H., Masulis, R.W., 1980. Optimal capital structure under corporate and personal taxation. Journal of Financial Economics 8, 3–29.

Debreu, G., 1959. Theory of Value. John Wiley & Sons, New York.

Dybvig, P., Ross, S., 1986. Tax clienteles and asset pricing. Journal of Finance 41, 751–771.

Elton, E.J., Gruber, M.J., 1978. Taxes and portfolio composition. Journal of Financial Economics 6, 399–410.

Gilson, S.C., Hotchkiss, E.S., Ruback, R.S., 1998. Valuation of bankrupt firms. Working paper, Harvard Business School.

Gordon, R.H., Varian, H.R., 1989. Taxation of asset income in the presence of a world securities market. Journal of International Economics 26, 205–226.

Hamada, R., 1969. Portfolio analysis, market equilibrium and corporation finance. Journal of Finance 24, 13–31.

Hamada, R.S., 1972. The effect of the firm's capital structure on the systematic risk of common stocks. Journal of Finance 27, 435–452.

Hamada, R.S., Scholes, M.S., 1985. Taxes and corporate financial management. In: Edward, I.A., Subrahmanyam, M.G. (Eds.), Recent Advances in Corporate Finance. Richard D. Irwin, Homewood, IL.

Kaplan, S.N., Ruback, R.S., 1995. The valuation of cash flow forecasts: An empirical analysis. Journal of Finance 50, 1059–1093.

Litzenberger, R.H., Ramaswamy, K., 1979. The effect of personal taxes and dividends on capital asset prices. Journal of Financial Economics 7, 163–195.

Long Jr., J.B., 1977. Efficient portfolio choice with differential taxation of dividends and capital gains. Journal of Financial Economics 5, 25–53.

Miller, M.H., 1977. Debt and taxes. Journal of Finance 32, 261–275.

Modigliani, F., Miller, M.H., 1963. Corporate income taxes and the cost of capital: A correction. American Economic Review 53, 433–493.

JBF 1769
26 April 2002 Disk used
ARTICLE IN PRESS
No. of Pages 16, DTD=4.3.1
SPS, Chennai

Myers, S.C., Ruback, R.S., 1992. Discounting rules for risky assets, unpublished working paper.

Ross, S., 1987. Arbitrage and martingales with taxation. Journal of Political Economy 95, 371–393.

Ross, S.A., Westerfield, R.W., Jaffee, J.F., 1993. Corporate Finance. Irwin, Homewood, IL.

Ruback, R.S., 1986. Calculating the market value of riskless cash flows. Journal of Financial Economics 15, 323–340.

Schaefer, S., 1982. Taxes and security market equilibrium. In: William, S., Cootner, C. (Eds.), Financial Economics: Essays in Honor of Paul Cootner. Prentice Hall, Englewood Cliffs, NJ.

Sick, G.A., 1990. Tax-adjusted discount rates. Management Science 36, 1432–1450.

Singer, R.F., 1979. Endogenous marginal income tax rates investor behavior and the capital asset pricing model. Journal of Finance 34, 609–616.

Taggart, R.A., 1991. Consistent valuation and cost of capital expressions with corporate and personal taxes. Financial Management 20 (Autumn), 8–20.

Talmor, E., 1989. Tax arbitrage restrictions and financial leverage clienteles. Journal of Banking.