Valuing employee stock options and restricted stock in the presence of market imperfections*

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ABSTRACT

We develop a new technology for valuing financial assets such as employee stock options and restricted stock. To value such assets we use a model that takes explicit account of the non-diversification of the owner of the asset. The model is an extension of the common binomial pricing model and is relatively easy to implement. This paper explains the issues and uses a database of employee stock options to estimate the model parameters.

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1. Introduction

In this paper we introduce a simple model for the valuation of employee stock options (ESO) that takes account of market imperfections. The model, based on a paper by Benninga, Helmantel, Sarig (2005, henceforth BHS) can account for market imperfections such as vesting and non-marketability. It is easy to implement in a binomial framework. A simple extension of the model can also be used to value restricted stock units (RSU).

2. A review of the binomial model

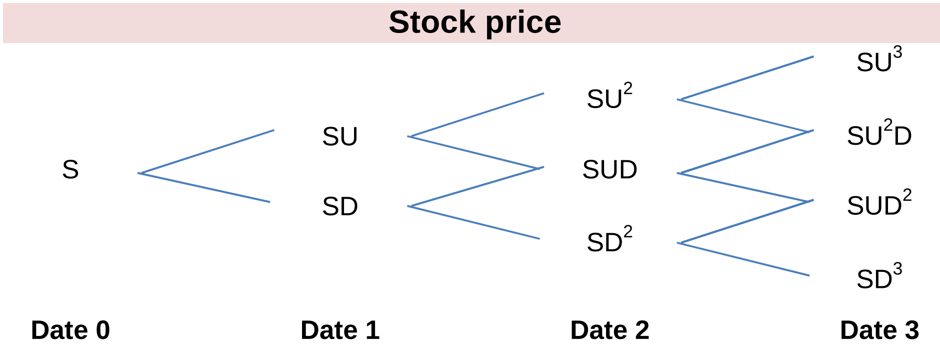
The binomial model is, after the Black-Scholes model, the best-known option pricing model. In this section we review this model. The advantages of the binomial model are its simplicity, its easy computability, and (under appropriate assumptions) its congruence with the Black-Scholes framework.

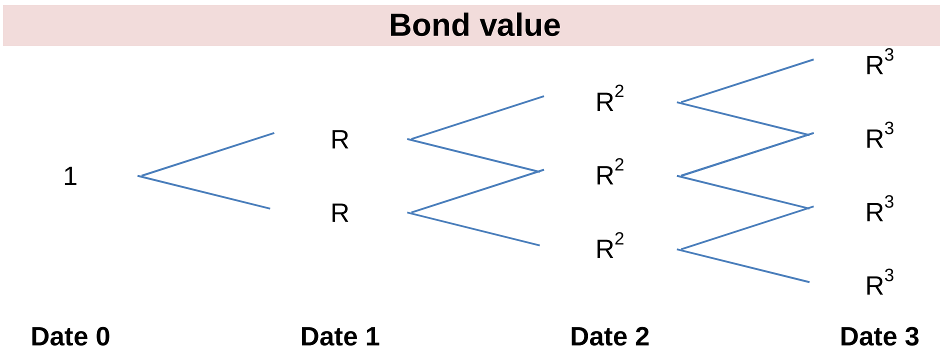
Suppose we are trying to price an option on a stock. Suppose further that the option has exercise price X and that the current price of the stock is S. We divide the interval (0,T) into n subintervals of length Δt: $\{0, \Delta t, 2\Delta t, ..., n\Delta t = T\}$. The binomial model assumes that in each time period Δt the price of the underlying asset either goes up by a factor U or down by factor D.[1] Thus at time Δt is the stock price is either *SU* or *SD*. By recursion, the price of the underlying asset at time jΔ is one of the following prices:

$\{SU^j, SU^{j-1}D, SU^{j-2}D^2, ..., SD^j\}$.

The graphical version of this model is well-known:

[1] U, D, and R are related to the size of the interval Δt, but for simplicity we have repressed this relationship in our notation. For completeness: If U and D are derived from a lognormal process with annual mean μ and standard deviation σ, then $U = \exp\left[\mu\Delta + \sigma\sqrt{\Delta t}\right]$, $D = \exp\left[\mu\Delta - \sigma\sqrt{\Delta t}\right]$, and $R = \exp\left[r\Delta t\right]$.





State prices in a binomial model

At any time jΔt there are two future states of the world, typified by the two possible stock prices S_jU or S_jD. The state price q_U is the per-dollar discount factor for dollars in the "Up" state (where the stock price is S_jU) and the state price q_D is the per-dollar discount factor for the other state. Graphically:

It can be shown that

$$q_U = \frac{D-R}{R(U-D)} \; , \quad q_U = \frac{U-R}{R(U-D)} \; .$$

Given the state prices, we can easily price European or American options. For example, a European call and put on the stock with expiration *T* and strike X can be priced by:

$$Call = \sum_{i=0}^{n} \binom{n}{i} q_U^i q_D^{n-i} Max\left(SU^iD^{n-i} - X, 0\right)$$

$$Put = \sum_{i=0}^{n} \binom{n}{i} q_U^i q_D^{n-i} Max\left(X - SU^iD^{n-i}, 0\right)$$

American options can also be priced by using the state prices.[2]

[2] See Benninga, *Financial Modeling*, 2008, Chapter 19 for details.

2. Public versus private state prices

Our model is based on the assumption that state prices depend on whether an individual can freely dispose or buy the asset under question. If an individual operates in competitive markets, then the asset is priced by "public state prices" $q_U^{public} = \frac{R-D}{R(U-D)}$ and $q_D^{public} = \frac{U-R}{R(U-D)}$ as defined in the previous section. If, however, the holder of the asset is restricted in selling the asset, we assume that a different set of state prices holds. We call these prices the "private state prices" and assume that

$$q_U^{private} = q_U^{public} - \delta$$
$$q_D^{private} = q_D^{public} + \delta$$

In this paper we do not formally derive this model. Instead we discuss it informally.[3]

Why do private state prices differ from public prices

The most accessible way to think about public versus private state prices is to consider a numerical example. Suppose that Δt = 1 (meaning that we consider annual payoffs and returns) and that $q_U^{public} = 0.3,\ q_D^{public} = 0.6$. This means that an asset paying off 1 in the Up state and 0 in the Down state is valued at 0.3 today and that an asset paying off 0 in the Up and 1 in the Down is valued at 0.6 today. It also means that the interest rate is 11.11%:

$$r = \frac{1}{R} - 1 = \frac{1}{q_U^{public} + q_D^{public}} - 1 = \frac{1}{0.9} - 1 = 11.11\%$$

Now suppose that, following our model, restricted assets are priced by private state prices. If δ = 0.02 then:

$$q_U^{private} = 0.3 - \delta = 0.28$$
$$q_D^{private} = 0.6 + \delta = 0.62$$

Then restricted assets that pay off in the Up state are worth less than if they were traded and assets that pay off in the Down state are worth more. This means that a call option is worth less in the "private market" than in the "public market."[4]

The intuition behind the private state prices is that an individual who holds restricted assets—be they stock or employee stock options—suffers from non-diversification, and that this non-diversification expresses itself in the individual having a more-than-optimal amount in Good states and less-than-optimal amount in Bad states of the world. Consider, for example, an employee in XYZ Corp. who has been given employee stock options in XYZ as part of her compensation package. Since the employee's wages are already tied to the fortunes of XYZ, the addition of (restricted) stock options further increases the employee's dependence on the company. If the employee were allowed to optimally diversify by

[3] For a formal model and derivation, see BHS (2005) and Abudy and Benninga (2010).
[4] Of course there is no "private market," but we use this as convenient shorthand.

selling the options, she would purchase assets that are negatively correlated with the fortunes of XYZ, paying off more if XYZ ended in a Bad state of the world.

We express this non-optimal diversification of the ESO holder by assuming that her private state prices value the Good state of the world less than the public state prices and that the private state prices value the Bad state of the world more than the public state prices. This is what is meant by the conditions $q_U^{private} = q_U^{public} - \delta,\ q_D^{private} = q_D^{public} + \delta$. The fact that $q_U^{private} + q_D^{private} = q_U^{public} + q_D^{public}$ is meant to express the assumption that the non-diversified individual has the same access to borrowing/lending markets as the diversified individual.[5]

Private state prices lead to early call option exercise

A remarkable result of our public vs private pricing model is that an individual who prices options using his private state prices will find it optimal to early-exercise a call option. To frame this result, recall that a standard result in option pricing theory is that the market price of call option on a non-dividend paying stock is always higher than the option's intrinsic value.[6] If, however, the individual prices an option using private state prices, this result no longer holds, and there is always a point where the individual would prefer early exercise to continued holding of the option.

We have rigorously proved this assertion in another paper.[7] In this section we will indicate the rough outlines of the proof and provide a graphical example. Recall from section ??? that a

$$Call = \sum_{i=0}^{n} \binom{n}{i} \left(q_U^{Public}\right)^i \left(q_D^{Public}\right)^{n-i} Max\left(SU^i D^{n-i} - X, 0\right)$$

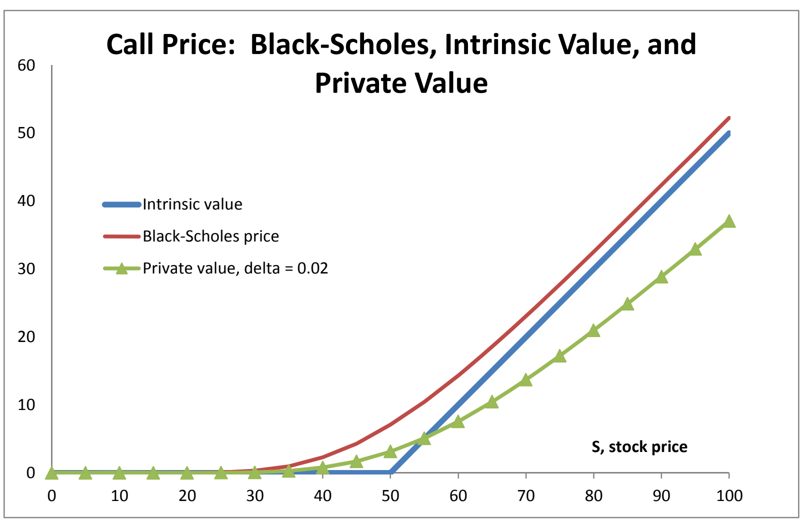
The spreadsheet below shows some sample calculations:

[5] The assumption of equal access can be weakened, but this does not materially affect our theoretical conclusions.
[6] This result is so well-known that it really requires no reference, but the interested reader can confirm it in Hull (2008).
[7] Abudy and Benninga (2010)

	A	B	C
1	**SIMULATING PUBLIC AND PRIVATE CALL VALUATIONS**		
2	Stock price, S	80	
3	Option exercise, X	50	
4	Option maturity, T	0.75	
5			
6	**Stock price process**		
7	μ, mean	15%	
8	σ, standard deviation	35%	
9	r, risk-free	6%	
10	Δt, time interval	0.004	<-- =1/250
11			
12	Up	1.0230	<-- =EXP(mu*delta_t+sigma*SQRT(delta_t))
13	Down	0.9787	<-- =EXP(mu*delta_t-sigma*SQRT(delta_t))
14	R	1.0002	<-- =EXP(rf*delta_t)
15			
16	**Public state prices**		
17	q_U^{Public}	0.4862	<-- =(R_-Down)/(R_*(Up-Down))
18	q_D^{Public}	0.5135	<-- =1/R_-B17
19			
20	**Private state prices**		
21	δ, non-diversification effect	0.02	
22	$q_U^{Private}$	0.4662	<-- =B17-delta
23	$q_D^{Private}$	0.5335	<-- =B18+delta
24			
25	Public option price (~Black-Scholes)	32.5449	<-- =Binomial_eur_call(Up,Down,R_,B2,B3,ROUND(B4/delta_t,0),0)
26	Private option price	20.9808	<-- =Binomial_eur_call(Up,Down,R_,B2,B3,ROUND(B4/delta_t,0),delta)
27	Intrinsic	30.0000	<-- =MAX(B2-B3,0)

The public option price (cell B25) is very close to the Black-Scholes price.[8] In this particular example, the private option price is 20.98 (cell B26), which is less than the intrinsic value of 30 (cell B27). Thus—in the absence of the opportunity to sell the option—the holder of the option would prefer to exercise early.

When we graph the Black-Scholes price, the intrinsic value, and the private option price, we clearly see that the private price intersects the intrinsic value:



In the above drawing, this happens when S = 55.16. The operative conclusion:

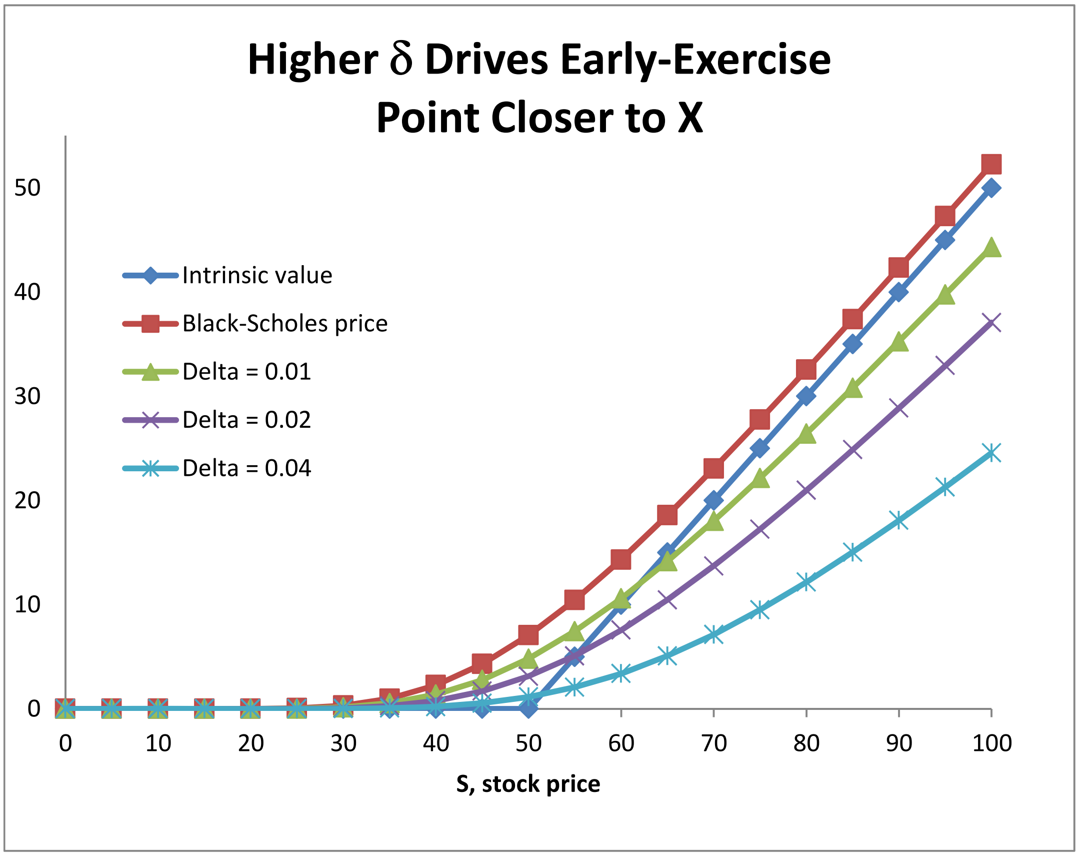
[8] When $\Delta t \to 0$, the binomial price using the public state prices converges to the Black-Scholes price. In our example $\Delta t = 0.004$, so that the binomial is a very good approximation to Black-Scholes.

If the holder of an ESO is past the vesting period and the stock price > 55.16, then early exercise of the ESO is optimal.

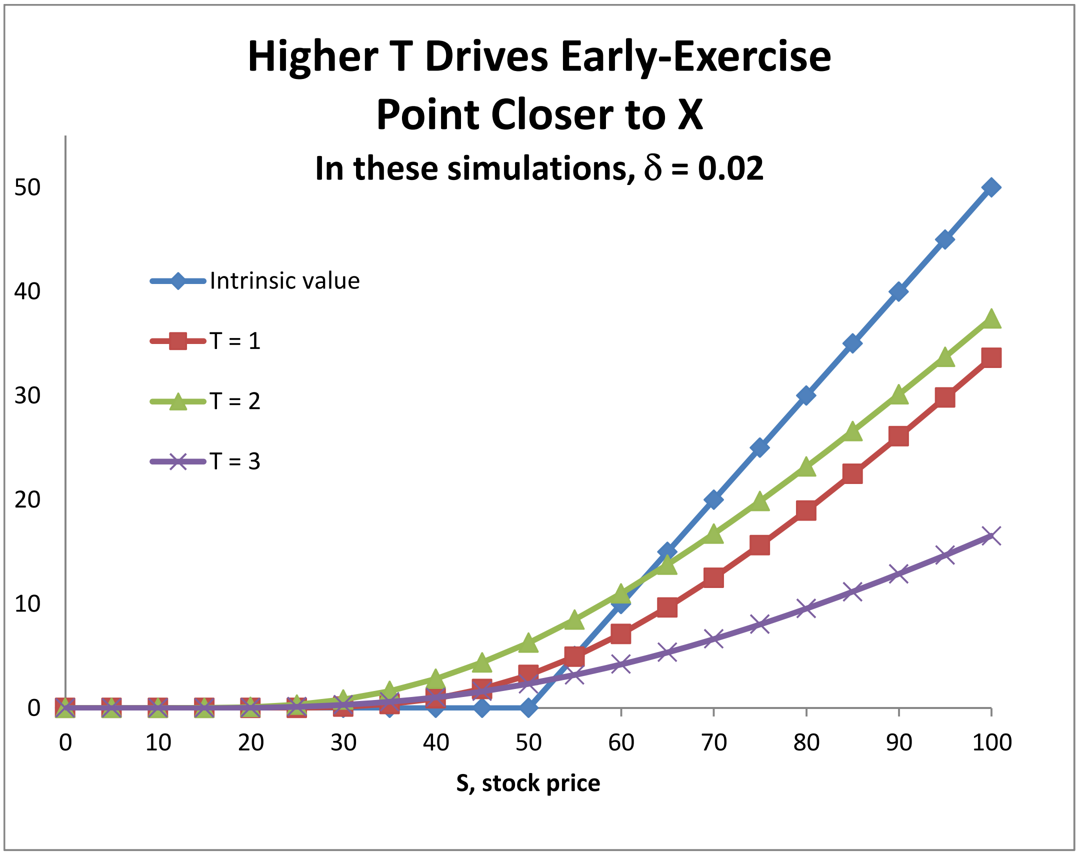
Sensitivity analysis

We can easily do sensitivity analysis on these computations to show the following:

- The crossing point (where intrinsic value = private value) is always greater than S = X.
- The crossing point is a decreasing function of δ. Larger δ implies that the crossing point is closer to S = X.



- The crossing point is a decreasing function of time to option maturity. The larger the option maturity, the closer is the crossing point to S = X.



3. Estimating the public versus the private state prices

In order to estimate the non-diversification measure δ we use a proprietary data set of employee stock option grants and exercise records. This proprietary data set that was obtained from Tamir-Fishman & Co., an Israeli based investment house which offers management services of share-based compensation programs. Tamir-Fishman supplied this data on the condition that the companies and their employees' identity remain anonymous. In this respect, we identify the companies by a two-digit code and report the results on an aggregate level using two-digit SIC code.

The Tamir-Fishman database is comprised of complete histories of stock option grants, vesting structures, option exercises and cancellation events for employees in both private and public firms. We identify ninety two firms in the database that are either currently public or were public in the past and private firms that were acquired by a public firm and now serve as its subsidiary granted SOPs. These firms are traded in Israel, U.S. and European stock markets.

We process the data according to the following criteria:

- Employees are sometimes forced to exercise their stock options. These forced exercises are usually results of job termination and mergers and acquisitions.[9] We are interested in voluntary

[9]Forced exercise due to job termination or as a result of mergers and acquisitions is a common practice. In case the company did not force early exercise and the employee exercised his option during 100 days from the job termination, we did not exclude his exercise records from our sample.

exercise records and therefore our aim is to exclude exercise records that represent forced exercise. As a result, we exclude all exercise records that were made 100 days before or after the employee's job termination date. The 100 day period relates to the common practice that allows employees up to three months to exercise their stock options after they cease working in the company. We exclude 100 days preceding the job termination date to account for the case that the employee exercises his options as part of his plan to cease working in the company.

- The database includes stock options that were granted to service providers that are not employees of the company. Companies use this compensation form to compensate service providers without paying cash. We are interested in valuing employee stock options, and therefore exclude exercise records of non-employees from our sample.
- In order to price stock options we need to estimate the standard deviation of the underlying stock—a crucial component in setting the value of the state prices as demonstrated in Section 2. We estimate the historical volatility and use it as a measure for the security standard deviation. As a result, we only relate to options that were granted while the underlying security was traded, and exclude exercise records in case the grant date occurred prior to the company's IPO
- An additional requirement with respect to the historical volatility calculation is a minimum of 14 trading days in a month. The reason is straightforward: Stocks with low trading volume underestimate the volatility measure. Therefore, we exclude exercise records in months that this criterion is violated.
- To refrain from bias in the estimation, we exclude exercise records in case a single exercise record resulted in less than 50 stocks.
- A stock option with an exercise price of zero is basically restricted stock, since the option holder does not pay any amount upon exercise. Thus, granting options with low exercise price is parallel to a restricted stock grant. Since our estimation method focuses on options, we exclude exercise records with exercise price lower than 0.1.
- We exclude ESO exercise records that were 100 days before the contractual expiration date of the option. As presented on Section 2, the private pricing model results in endogenous early exercise decision. Employees that exercise their options close to expiration are not suitable candidates for examining early exercise patterns and therefore cause to bias in the non-diversification estimation/

The final sample contains 33,294 employee-by-employee exercise records from 65 companies. The sample period of the stock option grants is between 1995 and 2009. The sample period of the exercise records is between 1998 and 2009.

ESO parameters and data sources

The Black-Scholes model and the binomial model require six input parameters: The underlying security price, the option's exercise price, expected standard deviation of the underlying asset, risk free rate, time to expiration and dividend yield.[10] We estimate the ESO value on the grant date and on the exercise date. For each ESO exercise record we match the following estimation:

- Historical volatility: We calculate the historical volatility of the underlying security using the daily continuous compounded return. We require a minimum estimation period of 20 trading

[10] ASC 718 (previously FAS 123(R)) also requires that these input parameters will be included in the valuation model of the equity based compensation.

days from the firm's IPO (subject to 14 trading days in a month restriction). We expand the estimation window to 30 days and then used a rolling window estimation of 30 trading days.[11] The proxy for the expected volatility is the historical volatility in the 'calculation date'. For example, if we calculate the ESO value on the grant date, then the historical volatility on the grant date serves as the proxy of the expected volatility in the pricing model.

- Risk free rate: We match a risk free rate according to the currency the company's shares are traded. For example, if the company's shares are traded in the NASDAQ we use the U.S. T-bill rate.
- Time to expiration: In our estimation we use the original expiration date of the ESO. The reason is that we wish to estimate the option on the grant date using the original parameters of the grant. Due to insufficient data, ESOs grants before 2000 were excluded from the sample. In addition, exercise records of ex-employees in which the original expiration date is identical to the last date of exercise were excluded.[12] We also excluded exercise records of ESO grants for less than four years.
- Dividend yield: We assumed a dividend yield of zero for the entire sample firms. This assumptions fits 80% of the sample firms during (and before) the sample period.
- Vesting period: We assume an average vesting period of three years for the entire sample. This assumption is relevant only to the estimation of ESO at the grant date.

The calculation of the above parameters involved the use of the following data sources: Stock prices were obtained from CRSP, Tel-Aviv stock exchange (TASE) website, Yahoo! Finance and websites of the companies in the sample. Annual risk free rates were obtained from the Federal Reserve Statistical Release website (3 months T-bill) and from websites of central banks in Israel and Europe, such as the Bank of Israel website (MAKAM rates).[13]

Descriptive statistics on the ESO parameters

In this subsection we provide summary of descriptive statistics for the sample companies and the model parameters. The objective is twofold: First, data on ESO grants in scarce in both academic and practice literature. Providing data about practices of this common equity base compensation form can be useful to policy makers. Second, we estimate the value of ESO using this data, and since these parameters eventually set the ESO value, it is important to describe its magnitude.

Table 1 provides a description of the companies industries according to the two-digit firm-level SIC codes as appears in CRSP. There is considerable heterogeneity in the firm industries type in the sample. In addition, a major part of the firms comprising the dataset are new-economy firms related to computers, software, the internet, telecommunications or networking.[14] These firms represent 69.23% of the firms and 73.83% of the exercise records in the sample.

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[11] We repeat our estimation using an historical volatility calculation of 126 trading days. The results are similar.

[12] The database is managed in a way that the expiration date changes according to the circumstances. For example, if the employee is no longer employed in the company, the expiration date is updated to three months after the job termination date. This update changes the estimation, so we exclude exercise records of grants that lack the original expiration date.

[13] MAKAM is a zero coupon bond issued by the Bank of Israel, parallel to one year Treasury bill.

[14] New economy firms defined as companies with primary SIC codes 3570, 3571, 3572, 3576, 3577, 3661, 3674, 4812, 4813, 5045, 5961, 7370, 7371, 7372 and 7373 (See Hall and Murphy 2003). As mentioned, we identify the companies using 2-digit SIC codes.

Table 1: Sample Firms Description			
Industry	Two-digit firm-level SIC	Percentage (number of firms)	Percentage (number of employees)
Food And Kindred Products	20	1.54%	0.16%
Paper And Allied Products	26	1.54%	0.73%
Printing, Publishing, And Allied Industries	27	1.54%	0.17%
Chemicals And Allied Products	28	3.08%	0.39%
Industrial And Commercial Machinery And Computer Equipment	35	16.92%	28.71%
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	36	26.15%	38.48%
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	38	7.69%	7.22%
Communications	48	7.69%	7.73%
Wholesale Trade-durable Goods	50	1.54%	0.20%
Depository Institutions	60	1.54%	2.09%
Business Services	73	26.15%	6.64%
Engineering, Accounting, Research, Management, And Related Services	87	4.62%	7.48%

Tables 2 and 3 present summary statistics on the original time to expiration of the options (in years) and on the remaining time to maturity of the options at the exercise date (in years), respectively. These characteristics are used to estimate the value of the nondiversification measure δ on the exercise date and the option value on the grant date. Table 2 presents a quite homogeneous picture: The average contractual option life ranges between eight to ten years, with some options grants for 16 years. The minimum period in the sample is four years. Combined with the data of Table 3, the data indicates that on average the ESOs in the sample are exercised when there are nearly two-thirds to half of the option term remaining. These findings are consistent with the findings of previous studies such as Huddart and Lang (1996) and Carpenter, Stanton and Wallace (2008). The sectors that deviate from this early exercise pattern are the food and kindred products and the paper and allied products (SIC codes 20 and 26, respectively).

Table 2: Time to maturity (in years) of the option						
Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	8.087	1.918	16.008	4.003	6.005	10.005
Food And Kindred Products	6.283	0.931	8.005	4.268	5.851	6.923
Paper And Allied Products	5.225	0.839	9.005	4.003	5.003	5.003
Printing, Publishing, And Allied Industries	10.008	0.001	10.008	10.005	10.008	10.008
Chemicals And Allied Products	10.023	0.039	10.181	10.005	10.005	10.008
Industrial And Commercial Machinery And Computer Equipment	7.162	1.571	10.507	4.123	6.003	8.003
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	9.091	1.511	16.008	4.003	7.164	10.008
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	5.785	1.628	10.008	4.003	5.000	5.003
Communications	9.751	0.572	10.008	5.849	10.005	10.005
Wholesale Trade-durable Goods	9.312	1.270	10.433	6.499	9.501	10.005
Depository Institutions	5.741	0.761	7.247	4.003	6.000	6.000
Business Services	9.032	1.844	10.008	4.003	9.871	10.008
Engineering, Accounting, Research, Management, And Related Services	6.938	0.679	10.008	4.044	7.003	7.005
Entire sample, employees	8.066	1.921	16.008	4.003	6.005	10.005
Entire sample, executives (Directors and Officers)	8.684	1.728	10.079	4.003	7.005	10.008
Entire sample, exercise (cash)	8.941	1.809	16.008	4.003	7.045	10.008
Entire sample, SDS (same day sale)	8.080	1.917	16.008	4.003	6.005	10.005

This table reports the time to maturity of the option grants at the grant date. The time to maturity is measured as the number of years between the grant date and the expiration date of the option grant. The summary statistics are computed over all the exercise records in the sample period. The summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Table 3: Remaining time to maturity of the options (in years) at the exercise date						
Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	4.669	2.334	9.978	0.274	2.871	6.564
Food And Kindred Products	1.868	0.770	2.975	0.529	1.104	2.555
Paper And Allied Products	1.527	0.768	4.003	0.288	0.852	1.979
Printing, Publishing, And Allied Industries	7.068	0.735	8.553	5.441	6.679	7.512
Chemicals And Allied Products	6.949	1.467	9.373	0.630	6.370	7.904
Industrial And Commercial Machinery And Computer Equipment	3.831	2.036	9.318	0.274	2.373	4.981
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	5.417	2.129	9.948	0.274	4.025	7.167
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	2.417	1.955	9.781	0.274	0.923	2.836
Communications	6.679	1.638	9.948	0.282	5.841	7.879
Wholesale Trade-durable Goods	8.493	1.093	9.663	5.975	8.600	8.992
Depository Institutions	3.101	1.317	6.197	0.395	1.962	4.196
Business Services	5.973	2.194	9.978	0.282	4.460	7.674
Engineering, Accounting, Research, Management, And Related Services	3.506	1.439	9.266	0.282	2.577	4.490
Entire sample, employees	4.630	2.333	9.978	0.274	2.830	6.460
Entire sample, executives (Directors and Officers)	5.749	2.115	9.748	0.477	4.013	7.504
Entire sample, exercise (cash)	5.238	2.399	9.660	0.282	3.586	7.274
Entire sample, SDS (same day sale)	4.664	2.333	9.978	0.274	2.866	6.554

This table provides the summary statistics over the sample period for the remaining term (in years) of the stock option at the exercise date. The remaining term is measured as the difference between the expiration date and the exercise date. The summary statistics is organized by the two-digit firm-level SIC categories ass reported by CRSP.

Table 4 reports the summary statistics of the stock price to the exercise price ratio in the sample. There is a difference in the ratios both across and within sectors. The highest ratios reflect run-ups in the stock market during our sample period. Specifically, these ratios stem from market run-ups during the end of the 1990s and the beginning of 2000. In general, the option exercise patterns present evidence on the persistence of early exercise behavior along with considerable heterogeneity both within and across sectors. These findings are consistent with the findings of previous studies such as Carpenter, Stanton and Wallace (2008) and Bettis, Bigjak and Lemmon (2005).

Table 4: The stock price to exercise price ratio at the exercise date						
Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	2.877	3.114	39.767	1.001	1.381	3.278
Food And Kindred Products	2.603	0.903	3.972	1.358	1.613	3.393
Paper And Allied Products	2.512	0.962	5.590	1.257	1.868	2.563
Printing, Publishing, And Allied Industries	2.964	0.671	4.339	1.654	2.619	3.500
Chemicals And Allied Products	1.958	0.600	5.152	1.012	1.485	2.359
Industrial And Commercial Machinery And Computer Equipment	3.339	3.314	39.767	1.006	1.551	4.091
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	2.871	3.311	37.758	1.001	1.281	3.414
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	1.892	1.250	30.415	1.010	1.368	1.869
Communications	2.393	2.577	28.980	1.009	1.548	2.525
Wholesale Trade-durable Goods	2.159	0.894	5.531	1.093	1.744	2.196
Depository Institutions	1.562	0.184	1.878	1.019	1.440	1.695
Business Services	3.926	4.476	26.158	1.001	1.471	4.242
Engineering, Accounting, Research, Management, And Related Services	2.128	0.822	8.281	1.028	1.580	2.395
Entire sample, employees	2.860	3.138	39.767	1.001	1.371	3.236
Entire sample, executives (Directors and Officers)	3.342	2.301	25.000	1.033	1.757	4.176
Entire sample, exercise (cash)	4.316	5.774	33.163	1.022	1.412	3.883
Entire sample, SDS (same day sale)	2.865	3.080	39.767	1.001	1.381	3.273

This table provides the summary statistics over the sample period for the ratio of the stock price to exercise price at the exercise date. The summary statistics is organized by the two-digit firm-level SIC categories ass reported by CRSP.

Estimation of the non-diversification measure δ and the ESO value

In this subsection we perform two estimations. Our purpose is to estimate the value of the ESO at the grant date. We use the following procedure: The first estimation occurs on the exercise date. We use the stock price on the exercise date, along with the additional option parameters such as the underlying security volatility, and estimate the non-diversification measure δ. Our second estimating occurs on the option's grant date. We calibrate the non-diversification measure from the first stage along with the stock option characters, and calculate the ESO value. This is basically the value of the stock option using the private pricing model. Then, we calculate a plain vanilla Black-Scholes option value on the grant date. Finally, we divide the private value by the Black-Scholes option value, and receive a measure of the value of the ESO to the employee on the grant date. Figure ?? summarizes the estimation procedure.

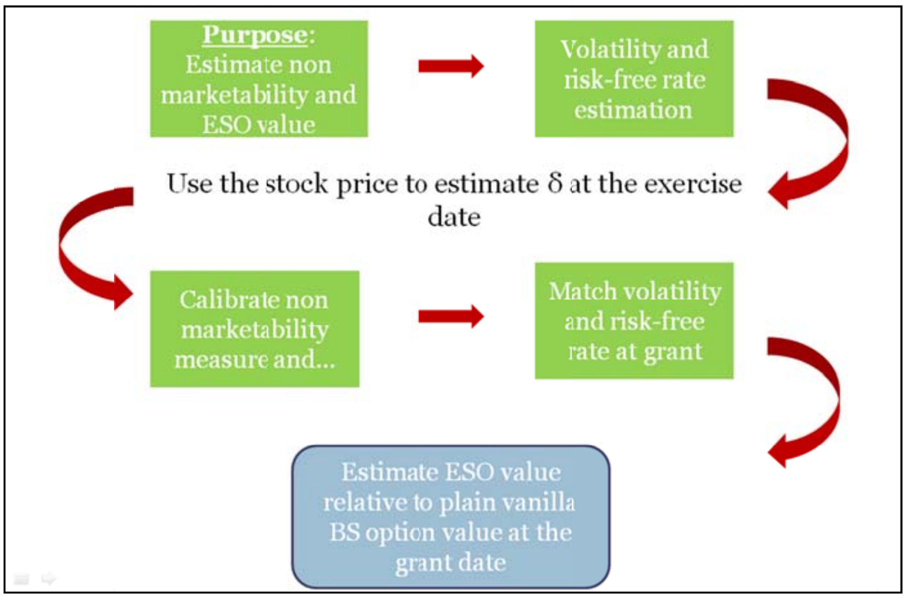


Table 5 presents the estimation results of the nondiversification measure δ using the procedure presented in Figure ??. This estimation is made on the (early) exercise date of the option, using the stock price on the exercise date along with the specific characters of each company, such as the risk free rate, volatility, dividend yield and exit rate.[15] In addition to the presentation according to CRSP SIC codes, an average data is calculated for rank and file employees relative to executives and to exercise records of employees that continue to hold the stock relative to exercise records in which the stock acquired is sold immediately (cashless exercise). The average nondiversification measure δ in the entire sample equals 0.025, with a similar tendency within the SIC sectors except for the food and kindred products and the paper and allied products (SIC codes 20 and 26, respectively). In addition, according to Table 5 results executives have a lower nondiversification measure relative to rank and file employees. It means that, ceteris paribus, executives tend to exercise their stock options later (or closer to expiration) than rank and file employees.

[15] We do not use vesting here because if the stock option can be exercised it means that it is after vesting.

Table 5: The non-marketability estimation using the stock price at the exercise date						
Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	0.025	0.036	0.474	0.000	0.005	0.030
Food And Kindred Products	0.009	0.007	0.027	0.002	0.004	0.014
Paper And Allied Products	0.010	0.006	0.035	0.003	0.007	0.011
Printing, Publishing, And Allied Industries	0.008	0.004	0.019	0.004	0.005	0.009
Chemicals And Allied Products	0.021	0.031	0.285	0.001	0.007	0.025
Industrial And Commercial Machinery And Computer Equipment	0.020	0.031	0.414	0.000	0.003	0.025
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	0.032	0.044	0.469	0.000	0.006	0.039
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	0.023	0.026	0.319	0.000	0.010	0.025
Communications	0.023	0.029	0.353	0.000	0.008	0.025
Wholesale Trade-durable Goods	0.030	0.028	0.160	0.004	0.016	0.028
Depository Institutions	0.026	0.027	0.299	0.007	0.015	0.026
Business Services	0.023	0.033	0.474	0.000	0.005	0.029
Engineering, Accounting, Research, Management, And Related Services	0.018	0.021	0.291	0.000	0.008	0.021
Entire sample, employees	0.026	0.036	0.474	0.000	0.005	0.031
Entire sample, executives (Directors and Officers)	0.016	0.021	0.328	0.000	0.004	0.022
Entire sample, exercise (cash)	0.027	0.042	0.328	0.000	0.004	0.032
Entire sample, SDS (same day sale)	0.025	0.036	0.474	0.000	0.005	0.030

This table reports the non-marketability estimation at the exercise date. We value the non-marketability using the spesific characthers of each exercise record. Time to maturity is measured as the number of years between the exercise date and the original expiration date of the option grant. Annual risk free rate is adjusted according to the share's currency. Volatility is estimated by historical volatility of the share. The summary statistics are computed over all the exercise records in the sample period and grouped using two-digit firm-level SIC categories as reported in CRSP.

Table 6 presents the estimation results. These estimations calibrate the non-diversification measure with the annual risk free rate, historical volatility, contractual option life, vesting period and dividend yield—all on the grant date—for both the ESO private value and the plain vanilla Black-Scholes value.

Table 6: ESO private value relaitive to Black-Scholes value (in percentage) at the grant date						
Industry	Average	SD	Max	Min	1st quartile	4th quartile
Full Sample	48.23%	29.62%	99.97%	0.00%	22.01%	74.24%
Food And Kindred Products	69.06%	16.17%	89.36%	32.05%	57.79%	82.24%
Paper And Allied Products	68.32%	13.21%	88.35%	25.40%	62.34%	76.02%
Printing, Publishing, And Allied Industries	65.30%	9.82%	76.42%	37.53%	60.05%	73.31%
Chemicals And Allied Products	50.35%	24.21%	93.75%	0.00%	31.24%	70.02%
Industrial And Commercial Machinery And Computer Equipment	57.97%	30.62%	99.97%	0.00%	29.92%	85.37%
Electronic And Other Electrical Equipment And Components, Except Computer Equipment	41.21%	30.37%	99.80%	0.00%	13.27%	70.14%
Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	47.15%	23.59%	98.87%	0.00%	32.45%	60.94%
Communications	43.73%	24.55%	99.17%	0.00%	28.47%	62.45%
Wholesale Trade-durable Goods	30.92%	18.55%	74.89%	0.00%	21.45%	39.14%
Depository Institutions	44.78%	17.30%	73.96%	0.00%	36.69%	56.42%
Business Services	48.47%	28.91%	99.46%	0.00%	26.03%	74.18%
Engineering, Accounting, Research, Management, And Related Services	50.92%	24.96%	97.57%	0.00%	33.22%	68.91%
Entire sample, employees	47.98%	29.68%	99.97%	0.00%	21.68%	74.05%
Entire sample, executives (Directors and Officers)	55.07%	27.00%	97.57%	0.00%	32.96%	78.73%
Entire sample, exercise (cash)	51.28%	32.34%	99.77%	0.00%	22.11%	80.45%
Entire sample, SDS (same day sale)	48.20%	29.59%	99.97%	0.00%	22.00%	74.20%

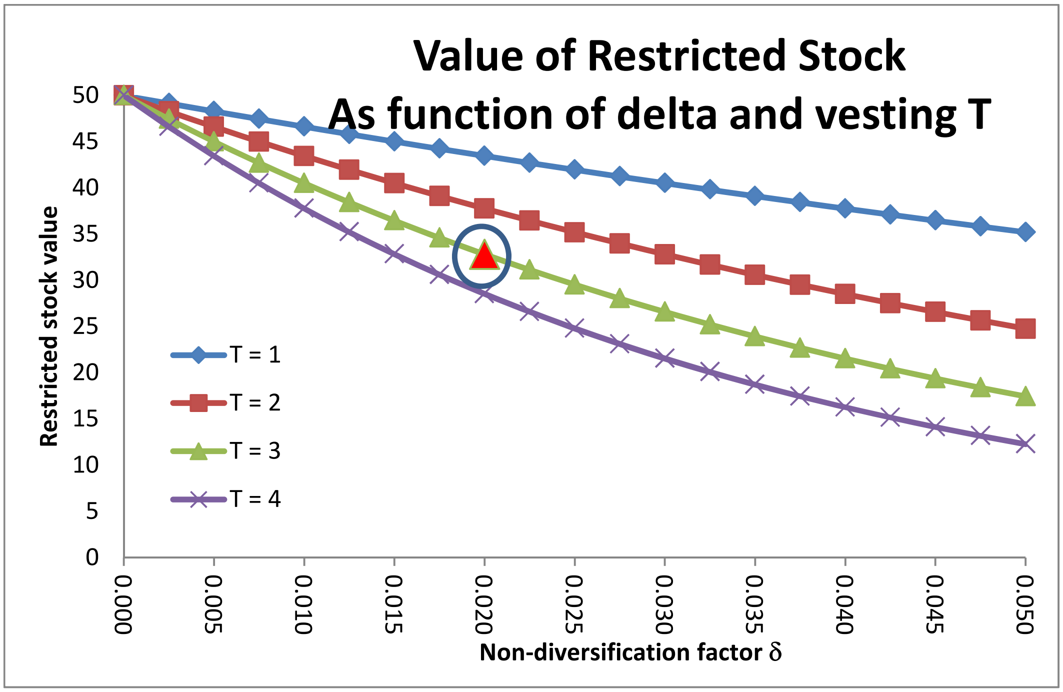
This table reports the value of the ESO using the private pricing model relative to a plain-vanilla Black-Scholes value of the stock option at the grant date. The non-marketability measure was estimatied at the exercise date and calibrated into the model. Time to maturity is measured as the number of years between the grant date and the original expiration date of the option grant. Annual risk free rate is adjusted according to the share's currency. Volatility is estimated by historical volatility of the stock. The summary statistics are computed over all the exercise records in the sample period, and grouped using two-digit firm-level SIC categories as reported in CRSP.

According to our estimation, the private ESO value is, on average, about 50% relative to a plain vanilla BS value. In the food and kindred products and the paper and allied products (SIC codes 20 and 26, respectively) industries the value is higher—around 65%. These findings are consistent with predictions of other academic papers regarding ESO value. For example Meulbroek (2001) predicts that in new economy firms (with exhibit higher stock volatility), an undiversified manager would assign lower value to his stock options relative to an un-diversified manager from less volatile industries. Our findings are also consistent with the findings of Ikaheimo et al. (2006), which use the prices of tradable executive stock options, traded at the Helsinki stock exchange after the options are vested (which means these are transferable stock options). By analyzing 27,808 trades, Ikaheimo et al. (2006) show major underpricing of the ESO which can reach over 50% discount relative to BS value. Since Ikaheimo et al. (2006) examine tradable stock options, the nonmarketability associated with these options should be less comparing to the standard case of untradeable stock options (which is the case of the stock options in Tamir-Fishman sample). It implies that the untradeable stock options the discount should be higher than the one found by Ikaheimo et al. (2006). In addition, the results ascribe higher option values to executives compared to ESO values to non-executive employees. Overall, these results point out on a relative high discount on equity based compensation.

4. Valuation of restricted stocks units

Restricted stocks are additional form of equity based compensation. In this compensation form, the employee is granted with either right to receive stocks once the vesting requirements are met or with stocks which are restricted until the vesting requirements are met. These compensation forms are called restricted stocks units (RSU) and restricted stock (RS), respectively, with RSU being the more common between the two. The key difference is that since RSU programs deliver the stocks to the employee only after the vesting period while RS programs deliver the (restricted) stocks at the grant date. Thus, RSU holders have no voting rights and usually are not entitled to receive dividend or dividend equivalents.

In this paper we use the private pricing model to value RSU. Since RSUs are not tradable only during the vesting period, we use private state prices with exogenous exit rate during the vesting period, (in which the stocks are non-tradable and the employee is subject to forfeit of the stocks upon job termination). After the vesting period, when the stock is tradable and unrestricted, we use public state prices without any other restrictions. Putting this differently, we basically discount the value of the stock on the vesting date and consider the non-marketability and forfeit restrictions while discounting. The following figure presents the value of a normalized restricted stock as a function of the non-marketability period for different values of δ.



The circled point shows that a restricted stock with a 3-year maturity and a non-diversification factor of δ = 0.02 is worth less than 70% of the value of a non-restricted stock.

5. Summary

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