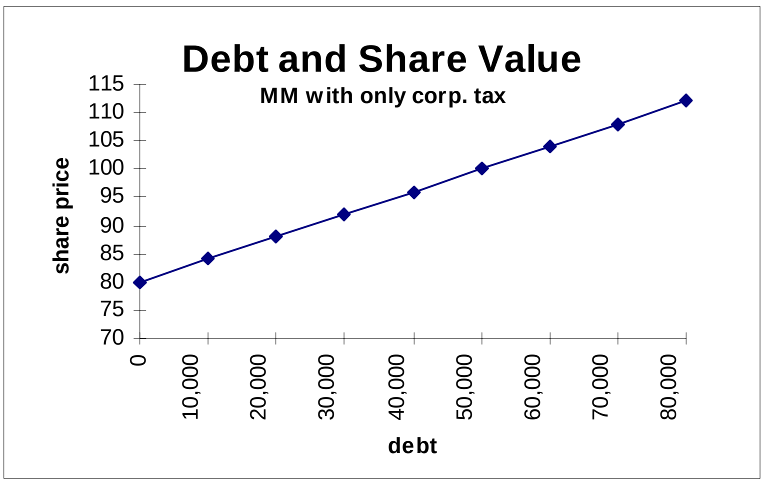
Modigliani-Miller with only corporate taxes:

$$V_L = V_U + \sum_{j=1}^{N} \frac{t_c * interest_j}{(1+i)^j}$$

If there is constant, infinitely-lived debt:

$$V_L = V_U + t_c D$$

This means: Share value increases linearly with debt:



$$r_e(L) = r(U) + [r(U) - r_d]\frac{D}{E}(1 - t_c)$$

$$WACC = \frac{FCF}{V(L)} \rightarrow WACC \downarrow \ as \frac{D}{E} \uparrow$$

Problems:

- Descriptive—what happens when I increase debt/equity ratio
- Not *normative*

Maybe MM made a mistake?

$$NPV(debt) = +debt - \sum_{j=1}^{N} \frac{(1-t_c)*interest_j}{(1+i)^j} - \frac{debt}{(1+i)^N}$$
$$= \sum_{j=1}^{N} \frac{t_c * interest_j}{(1+i)^j}$$

If, on the other hand, we discounted at the *after-tax debt rate*:

$$NPV(debt) = +debt - \sum_{j=1}^{N} \frac{(1-t_c)*interest_j}{\left(1+(1-t_c)\cdot i\right)^j} - \frac{debt}{\left(1+(1-t_c)\cdot i\right)^N}$$
$$= 0$$

However:

> Cash flows should be discounted at the discount rate of the *marginal purchaser* of the cash flows in the market.

So—if individuals are not taxed, then *the interest rate i* is the correct discount rate:

$$debt\ value = \sum_{j=1}^{N} \frac{interest_i}{(1+i)^j} - \frac{debt\ \ principal}{(1+i)^N}$$

I.e.:

> The IRR of the marginal bondholder from buying the bond at market prices is the bond's interest rate.

For the future (Miller model)

Suppose interest is taxed at rate t_d: Then

$$debt\ \ principal = \sum_{j=1}^{N} \frac{(1-t_d)interest_i}{(1+marginal\ \ bondh.\ \ IRR)^j} - \frac{debt\ \ principal}{(1+marginal\ \ bondh.\ IRR)^N}$$

$$\Rightarrow\ marginal\ \ bondholder's\ IRR = (1-t_d)i$$

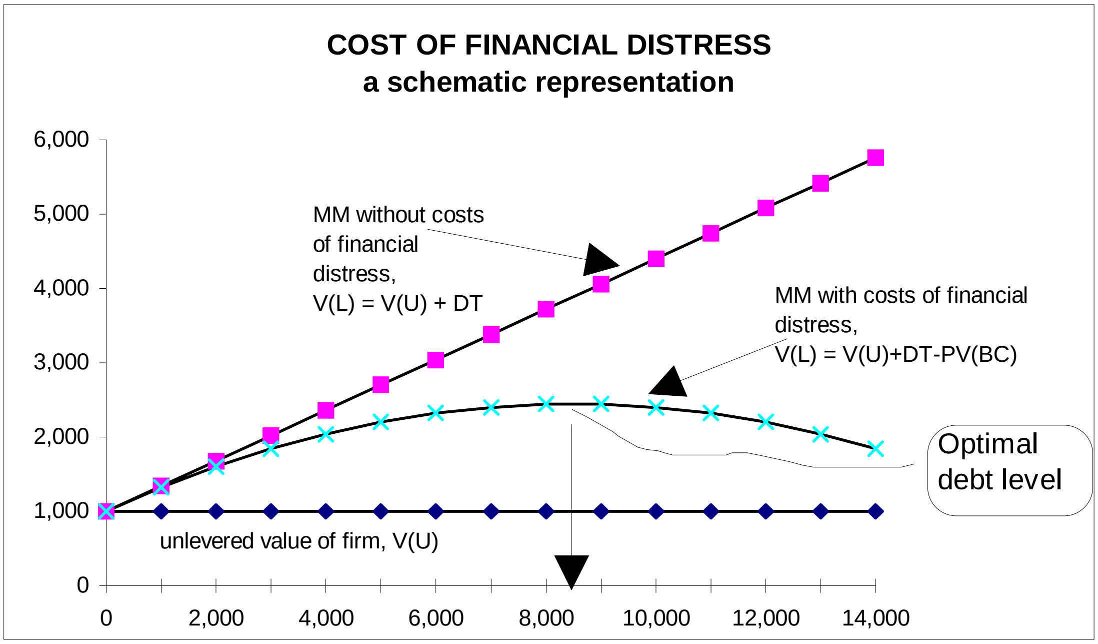
Other possibilities:

Costs of financial distress ("bankruptcy costs")

$$V_L = V_U + t_c D - PV(\textit{future costs of financial distress})$$

as popularly written:

$$V_L = V_U + t_c D - PV(\textit{Bankruptcy Costs})$$



Problem: Where are the bankruptcy costs?

Miller:

- "Terminal events of bankruptcy"
- Costly monitoring of covenants
- Costs of debt refinancing
- Costs of business disruption

"We dutifully acknowledged these well-known costs of debt finance, but we were hard put at the time to see how they could overweigh the tax savings of up to 50 cents per dollar of debt that our model implied.

"Not only did there seem to be potentially large amounts of corporate taxes to be saved by converting equity capital to tax-deductible interest debt capital, but there appeared to be ways of doing so that avoided, or at least drastically reduced, the secondary costs of high-debt capital structures.

" … Hybrid securities, such as income bonds … under which deductible interest payments could be made in the good years, but passed or deferred in the bad years without precipitating a technical default."

Source: Merton Miller, "The Modigliani-Miller Propositions after Thirty Years," *Journal of Economic Perspectives*, Fall 1988.

Option-like costs

Line of thought:

- In levered firm, equity-holders own a put option on the assets.
- Puts increase in value when the underlying security gets more risky.
- Thus:

  Trade-off between tax-benefits of leverage

  and increased riskiness of assets

Associated line of thought:

Debt gives shareholders more negotiating power.

"IF WE CAN'T EAT FROM THE TABLE, WE'LL KNOCK THE LEGS OFF."

Both of these theories (bankruptcy costs/option costs):

- Clearly true to some extent
- Hard to quantify
- Hard to translate to normative recommendations
- Difficult to see that they offset the tremendous tax advantage of MM (currently, $t_c \approx 40\%$, formerly $t_c \approx 50\%$).

Miller's "Debt and Taxes" Model

$$V_L = V_U + \sum_{j=1}^{N} \frac{\left[(1-t_d)-(1-t_c)\cdot(1-t_e)\right] * interest_j}{\left(1+(1-t_d)i\right)^j}$$

where:

- t_c is the corporate tax rate
- t_e is the marginal tax rate on equity earnings of the marginal equity purchaser
- t_d is the marginal tax rate on debt earnings (i.e., interest) of the marginal debt purchaser

Before we do the theory, some examples:

- Suppose $t_c = 40\%, t_d = 0\%, t_e = 0\%$. Then $(1-t_d)-(1-t_c)*(1-t_e) = 1-(1-t_c)*1 = t_c = 40\%$ i.e.:

  THE MILLER MODEL IS A GENERALIZATION OF THE MODIGLIANI-MILLER MODEL. THE MILLER MODEL TAKES INTO ACCOUNT MORE TAXES.

- Suppose: $t_c = 40\%, t_d = 39\%, t_e = 36\%$. Then

$$(1-t_d)-(1-t_c)*(1-t_e) = (1-39\%)-(1-40\%)*(1-36\%)$$
$$= 22.6\%$$

- Suppose: $t_c = 40\%, t_d = 39\%, t_e = 0\%$. Then

$$(1-t_d)-(1-t_c)*(1-t_e) = (1-39\%)-(1-40\%)*(1-0\%)$$
$$= 1\%$$

- Suppose: $t_c = 50\%, t_d = 70\%, t_e = 28\%$. Then

$$(1-t_d)-(1-t_c)*(1-t_e) = (1-70\%)-(1-50\%)*(1-28\%)$$
$$= -19\%$$

(“DuPont capital structure case”)

- Question: What’s the “marginal tax rate of the marginal equity holder on equity earnings”?

For Valuation Purposes

Suppose: $t_c = 40\%, t_d = 39\%, t_e = 10\%$. Then

$$(1-t_d)-(1-t_c)*(1-t_e) = 7\%$$

Is this enough to convince me that capital structure is important given:

- Uncertainty about discount rates
- Uncertainty about terminal values
- Uncertainty about taxes?
- Uncertainty about uncertainty?

Miller Model Theory

At *corporate level:* Advantage of corporate debt is its tax-deductibility.

At *personal level*:

- **Equity income** taxed at generally lower rate:

  capital gains exclusions

  postponability of taxation: IRAs, etc.

  tax evasion on dividends (Germany)

- **Debt income** taxed at higher rate:

  more difficult to avoid/postpone

  different clienteles from equity—hence different marginal tax rates.

Net advantage of debt over equity:

$$(1-t_d)-(1-t_c)(1-t_e)$$

Discount rate for this net advantage:

$$(1-t_d)\cdot i$$

(the marginal rate of the marginal purchaser of debt).

Advantages and disadvantages of Miller model:

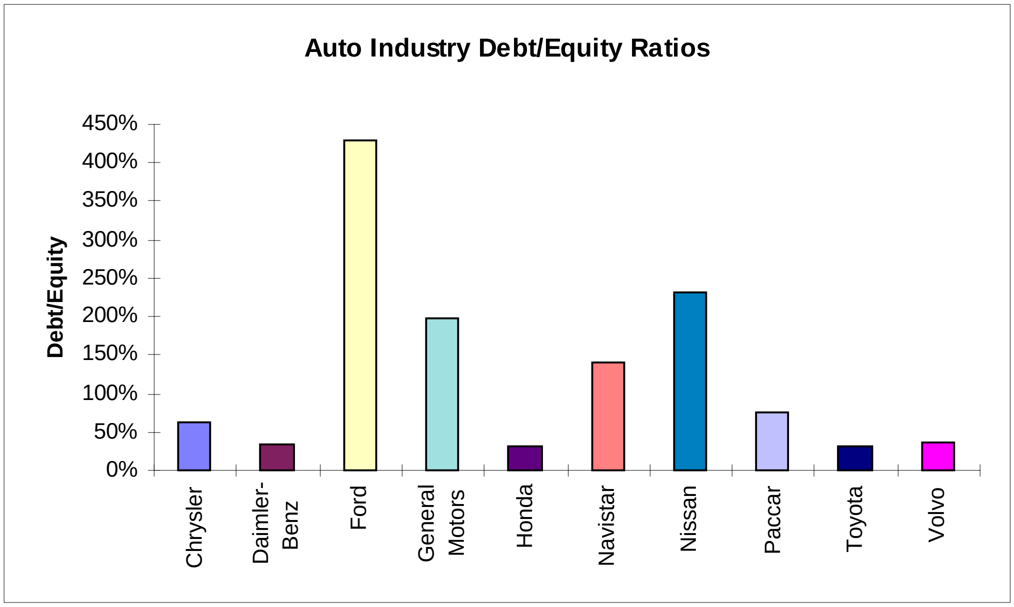
Advantages:

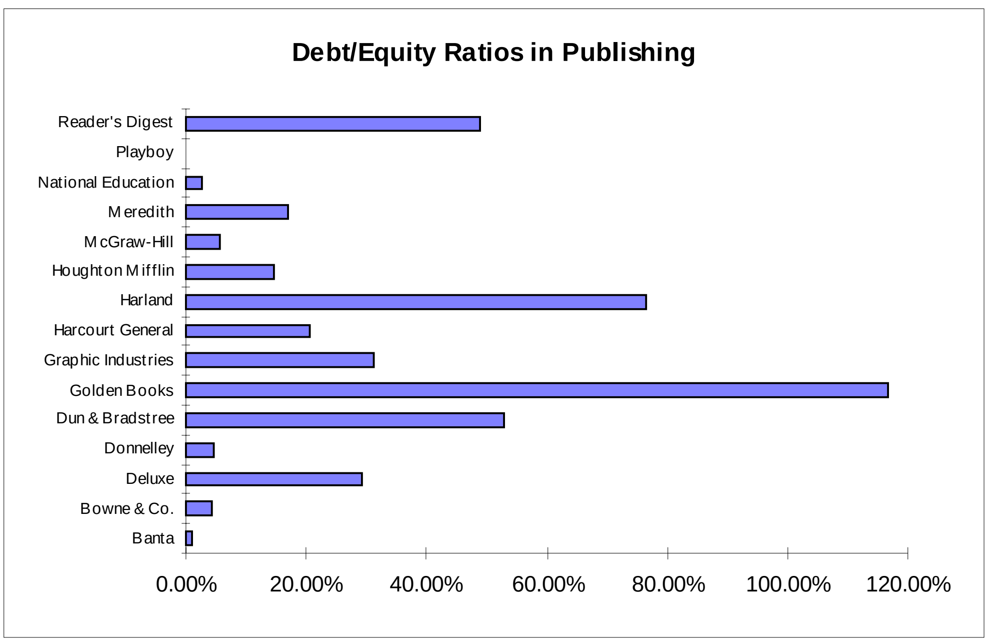
- Takes into account more complex taxation arrangements
- Can explain variety of debt/equity structures

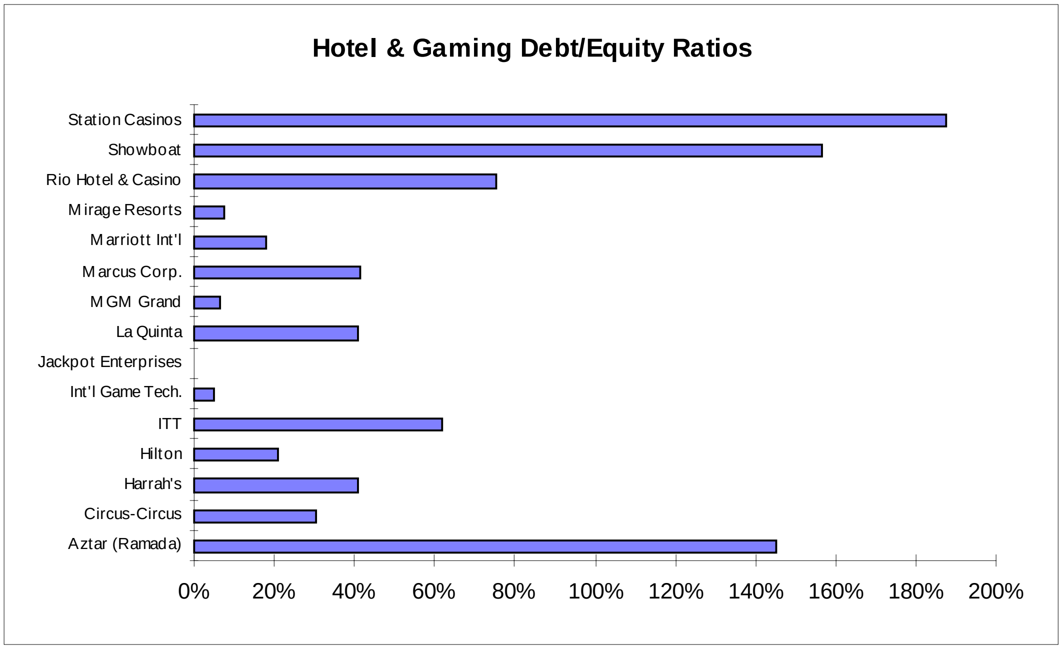
Disadvantages:

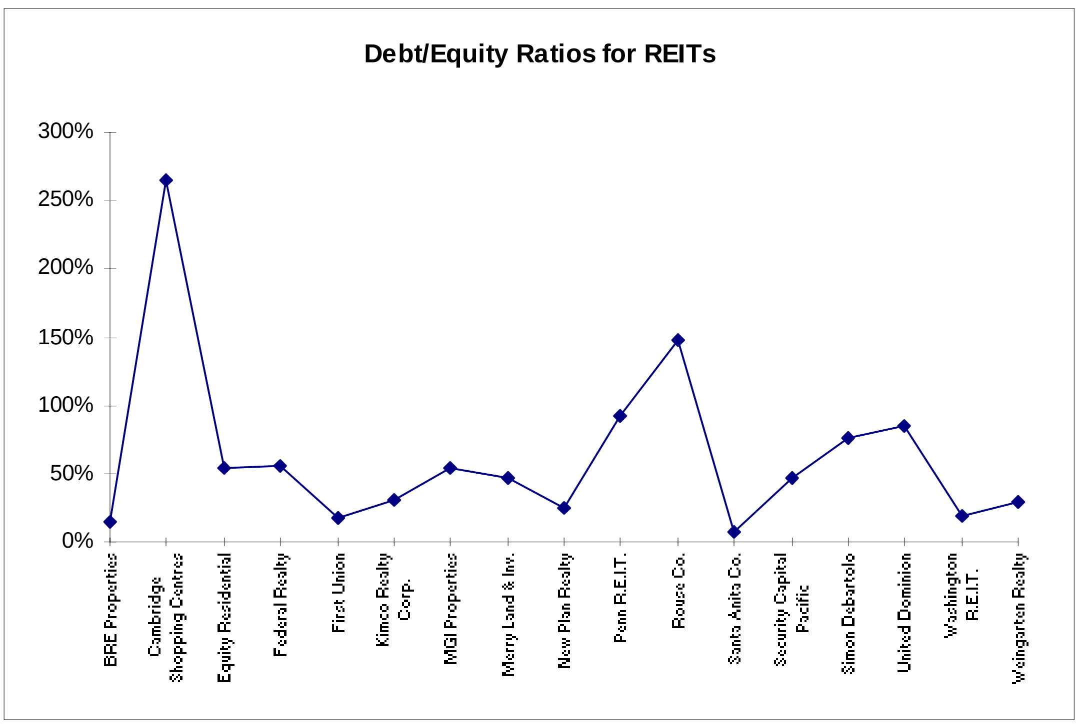
- Tax arbitrage—not a standard efficient markets theory—requires short-selling constraints
- Difficult to calculate the appropriate tax rates

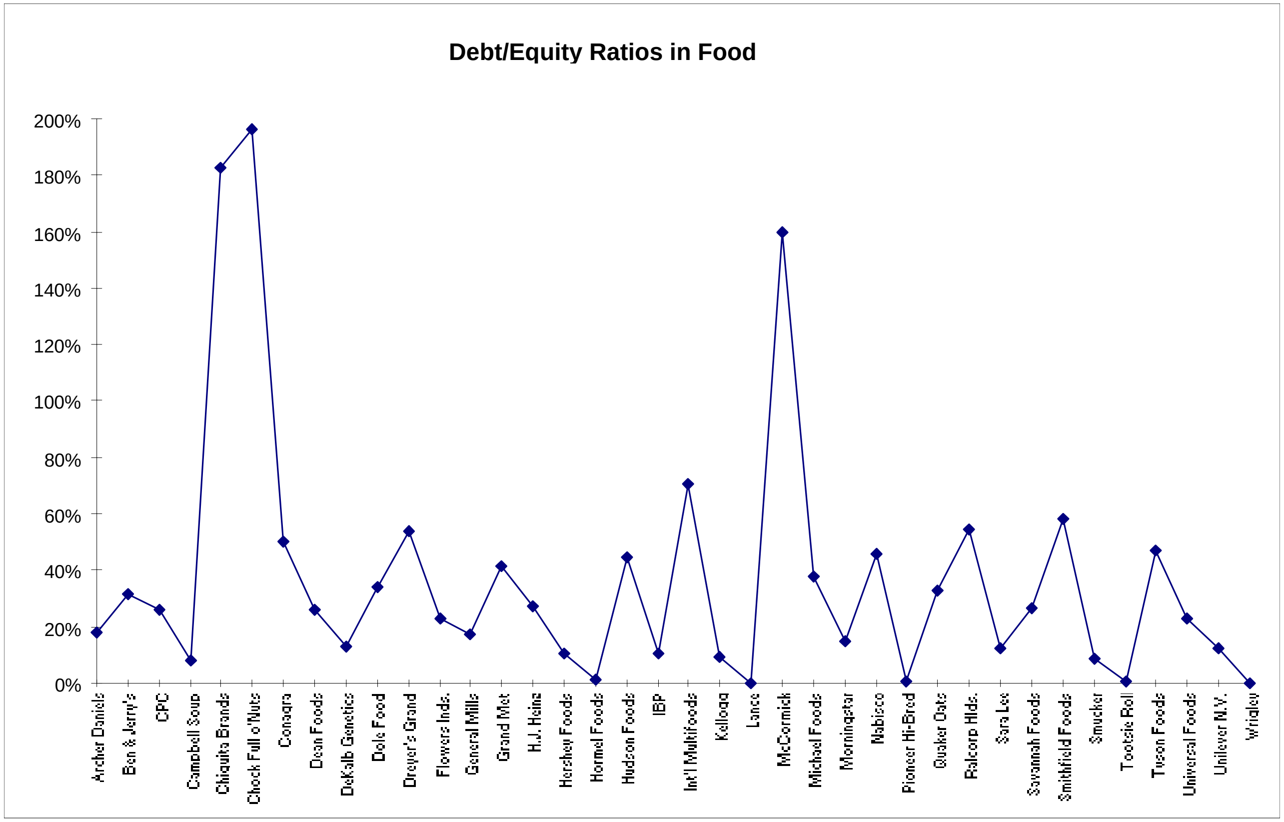
Are there debt/equity "norms" or "clustering" in industries as might be suggested by Modigliani-Miller?

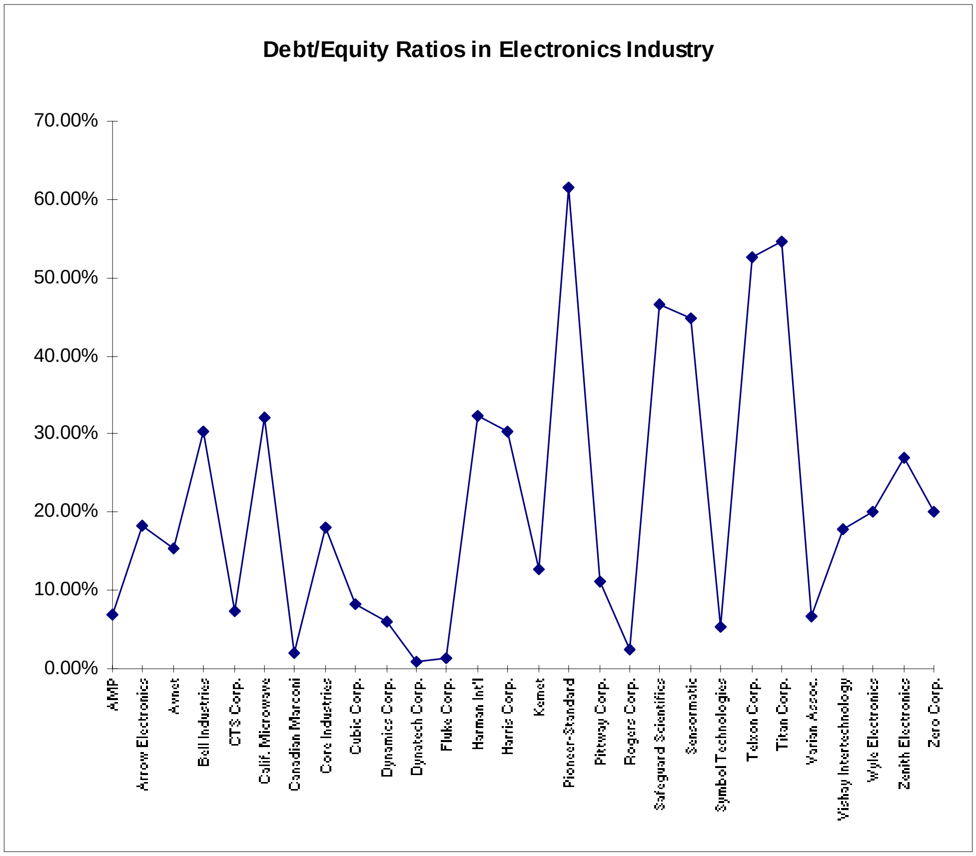





Hotel & Gaming Debt/Equity Ratios
Station Casinos
Showboat
Rio Hotel & Casino
Mirage Resorts
Marriott Int'l
Marcus Corp.
MGM Grand
La Quinta
Jackpot Enterprises
Int'l Game Tech.
ITT
Hilton
Harrah's
Circus-Circus
Aztar (Ramada)
0%
20%
40%
60%
80%
100%
120%
140%
160%
180%
200%


Debt/Equity Ratios for REITs
300%
250%
200%
150%
100%
50%
0%
BRE Properties
Cambridge Shopping Centres
Equity Residential
Federal Realty
First Union
Kimco Realty Corp.
MGI Properties
Merry Land & Inv.
New Plan Realty
Penn R.E.I.T.
Rouse Co.
Santa Anita Co.
Security Capital Pacific
Simon Debartolo
United Dominion
Washington R.E.I.T.
Weingarten Realty

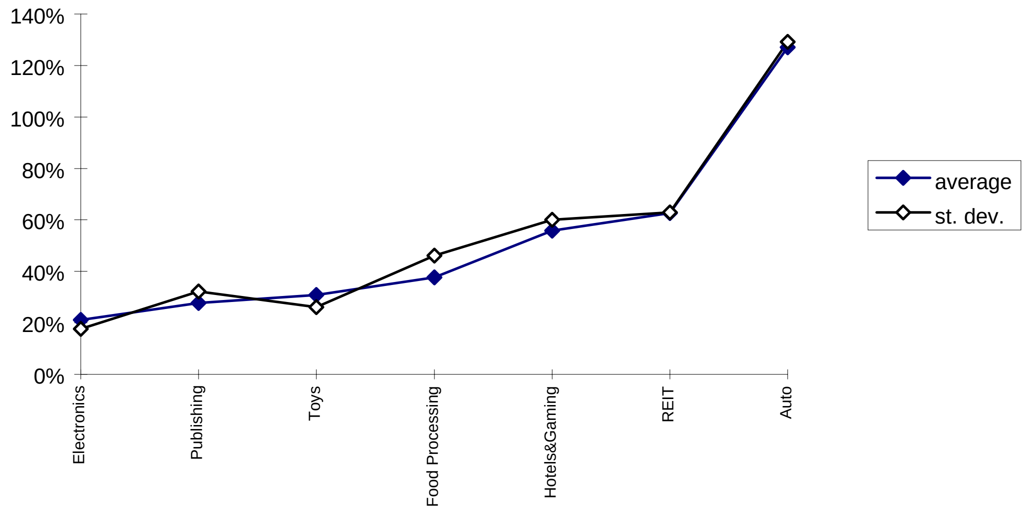

Debt/Equity Ratios in Food
200%
180%
160%
140%
120%
100%
80%
60%
40%
20%
0%
Archer Daniels
Ben & Jerry's
CPC
Campbell Soup
Chiquita Brands
Chock Full o'Nuts
Conagra
Dean Foods
DeKalb Genetics
Dole Food
Dreyer's Grand
Flowers Inds.
General Mills
Grand Met
H.J. Heinz
Hershey Foods
Hormel Foods
Hudson Foods
IBP
Int'l Multifoods
Kellogg
Lance
McCormick
Michael Foods
Morningstar
Nabisco
Pioneer Hi-Bred
Quaker Oats
Ralcorp Hlds.
Sara Lee
Savannah Foods
Smithfield Foods
Smucker
Tootsie Roll
Tyson Foods
Universal Foods
Unilever N.V.
Wrigley



Note: All data from *Value Line*, 1997. Debt/Equity ratio defined as:

$$\frac{\textit{Value Line's reported "Total Debt"}}{\#shares * current\ share\ price}$$

Debt/Equity--Summary Data



Miller Model—Infinitely-Lived Debt

Assumptions:

- Firm maintains constant debt level D
- $interest_t = i*D$

Then the Miller formula for V_L becomes:

$$
\begin{aligned}
V_L &= V_U + \sum_{j=1}^{N} \frac{\left[(1-t_d)-(1-t_c)\cdot(1-t_e)\right]*interest_j}{\left(1+(1-t_d)i\right)^j} \\
&= V_U + \sum_{j=1}^{N} \frac{\left[(1-t_d)-(1-t_c)\cdot(1-t_e)\right]*i*D}{\left(1+(1-t_d)i\right)^j} \\
&= V_U + \frac{\left[(1-t_d)-(1-t_c)\cdot(1-t_e)\right]*i*D}{(1-t_d)i} \\
&= V_U + \frac{\left[(1-t_d)-(1-t_c)\cdot(1-t_e)\right]}{(1-t_d)} D = V_U + TD
\end{aligned}
$$

Values of *T*?

These are the cases considered previously. Recall that:

$$T = \frac{\left[(1-t_d)-(1-t_c)\cdot(1-t_e)\right]}{(1-t_d)}.$$

- MM with only corporate taxes:
  $t_c = 40\%, t_d = 0\%, t_e = 0\%$ ➔ *T = 0.4.*

- Suppose: $t_c = 40\%, t_d = 39\%, t_e = 36\%$. ➔ *T = 0.37.*

- Suppose: $t_c = 40\%, t_d = 39\%, t_e = 0\%$. ➔ *T = 0.017.*

- Suppose: $t_c = 50\%, t_d = 70\%, t_e = 28\%$. ➔ *T = -0.2*
  (“DuPont capital structure case”)

- Suppose: $t_c = 40\%, t_d = 39\%, t_e = 10\%$. ➔ *T = 0.115.*

Miller's "Debt and Taxes":

Miller's ultimate claim:

IN EQUILIBRIUM T = 0.

REASONING

- **Who invests in equity?** For same risk class and expected return, individuals with *high* personal tax rates. Since $t_e < t_d$.

- **Who invests in bonds?** Individuals with *low* personal tax rates (all other things being equal).

- Denoting by r_{equity} the return on a typical equity security and by r_{debt} the return on a typical debt security (same risk, return), in equilibrium, at the margin:

$$r_{equity}(1-t_e) = r_{debt}(1-t_d) \Rightarrow r_{equity} = r_{debt}\frac{(1-t_d)}{(1-t_e)}$$

- If $T > 0$, corporations will issue more debt to raise value

  ➔ will raise interest i paid on debt

  ➔ will attract investors to bonds who have higher personal tax rates t_e and t_d.

- This increase in interest i makes debt relatively more attractive, and it *raises* the marginal tax rate t_d in the numerator of T. Looking at the previous expression:

$$\underset{\substack{\textit{stays}\\ \textit{same}}}{\underset{\uparrow}{r_{equity}}} \quad = \quad \underset{\textit{increases}}{\underset{\uparrow}{r_{debt}}} \quad \underset{\substack{\textit{must}\\ \textit{decrease}}}{\underset{\uparrow}{\frac{(1-t_d)}{(1-t_e)}}}$$

The effect on T:

$$T = \frac{(1-t_d)-(1-t_c)\cdot(1-t_e)}{(1-t_d)} = 1-(1-t_c)\left\{\frac{(1-t_e)}{(1-t_d)}\right\}\uparrow$$

$$\Rightarrow T\downarrow$$

$$(\textit{Note}:\ \left\{\frac{(1-t_e)}{(1-t_d)}\right\}\uparrow \ \textit{because from previous formula}\ \left\{\frac{(1-t_d)}{(1-t_e)}\right\}\downarrow\ )$$

In equilibrium (you guessed it!): $T = 0$.

SUPPOSE MILLER IS RIGHT and *T = 0*

- $$V_L = V_U + T * D = V_U,$$
  $$since \quad T = \frac{(1-t_d)-(1-t_c)(1-t_e)}{(1-t_d)} = 0$$

- WACC = *r(*U)

- $$r_e(L) = r(U) + \left[r(U)(1-T) - r_d(1-t_c)\right]\frac{D}{E}$$
  $$= r(U) + \left[r(U) - r_d(1-t_c)\right]\frac{D}{E}$$

  * notice that the first line is always true, even if $T \neq 0$.

  * notice that this formula agrees with the same formula for MM with only t_c.

- *WACC* is the IRR of the FCFs necessary to give *V(U)*.

$$-V(U) + \sum_{t=1}^{\infty}\frac{FCF_t}{(1+WACC)^t} = 0$$

Note that in all cases the WACC is the IRR of the FCFs necessary to give *V(L)*:

$$-V(L)+\sum_{t=1}^{\infty}\frac{FCF_t}{(1+WACC)^t}=0$$

Note that none of this is trivial. For example:

EVEN IF MILLER IS CORRECT, TO CALCULATE THE WACC FROM AN OBSERVED $R_E(L)$, WE MUST *UNLEVER* THE $R_E(L)$. (We discuss this and other computational topics in Chapter 9.)

MILLER and CAPM

- Calculating the **cost of equity in CAPM**, using the Miller model:

$$E\left(r_{equity}\right) = rf_{debt}\left(1 - t_c\right) + \beta_{equity}\left[E\left(r_{m,equity}\right) - rf_{debt}\left(1 - t_c\right)\right]$$

where

$$\beta_{equity} = \frac{Cov\left(return_{equity}, return_{m,equity}\right)}{\sigma^2_{\text{equity market portfolio}}}$$

$$= \frac{Cov\left(return_{equity}, return_{\text{equity market portfolio}}\right)}{\sigma^2_{\text{equity market portfolio}}}$$

(*this is the "regular" β*)

- Calculating the **cost of debt in CAPM**, using the Miller model:

$$E\left(r_{debt}\right) = rf_{debt} + \beta_{debt}\left[E\left(r_{m,equity}\right) - rf_{debt}\left(1 - t_c\right)\right]$$

where

$$\beta_{debt} = \frac{Cov\left(return_{debt}, return_{\text{equity market portfolio}}\right)}{\sigma^2_{\text{equity market portfolio}}}$$

(*note that the debt beta is calculated with respect to an equity market portfolio*)

- Calculating the **WACC in CAPM**, using the Miller model:

$$WACC = rf_{debt}(1-t_c) + \beta_{assets}\left[E(r_{m,equity}) - rf_{debt}(1-t_c)\right]$$

where

$$\beta_{assets} = \frac{E}{D+E}\beta_{equity} + \frac{D}{D+E}\beta_{debt}(1-t_c)$$

PROOF OF CAPM

Recall that:

$$r_{equity} = r_{debt}\frac{(1-t_d)}{(1-t_e)}$$

1. If Miller is correct and $T = 0$, then

$$T = \frac{(1-t_d)-(1-t_c)(1-t_e)}{(1-t_d)} = 0 \Rightarrow (1-t_c) = \frac{(1-t_d)}{(1-t_e)},$$

so that we can write:

$$r_{equity} = r_{debt}(1-t_c).$$

2. This means that for a *zero* β asset,

$$E\left(zero\,\beta\ equity,\ r_{equity}\right)*(1-t_e) =$$

$$E\left(zero\,\beta\ debt, r_{debt}\right)*(1-t_d) = rf_{debt}(1-t_d)$$

$$\Rightarrow E\left(zero\,\beta\ equity,\ r_{equity}\right) = rf_{debt}\,\frac{(1-t_d)}{(1-t_e)} = rf_{debt}(1-t_c)$$

Thus the intercept for the SMLs is:

$$debt\ SML\ intercept = rf_{debt}$$

$$equity\ SML\ intercept = rf_{debt}\left(1-t_c\right)$$

3. Write the equity SML as:

$$E\left(r_{equity}\right) = rf_{debt}\left(1-t_c\right) + \beta_{equity}\left[E\left(r_m\right) - ?\right]$$

Since the expected return of an asset with $\beta_{equity} = 1$ must be $E(r_m)$, it follows that ? = $rf_{debt}(1\text{-}t_c)$.

4. From the principle that $r_{equity} = r_{debt}\left(1-t_c\right)$, take $E(r_{debt})$ as it would be calculated by the *equity SML* and divide by $(1\text{-}t_c)$:

$$E\left(r_{debt}\right) = \frac{rf_{debt}\left(1-t_c\right) + \beta_{debt}\left[E\left(r_m\right) - rf_{debt}\left(1-t_c\right)\right]}{\left(1-t_c\right)}$$

$$= \frac{rf_{debt}\left(1-t_c\right) + \dfrac{Cov\left(r_{debt}\cdot\left(1-t_c\right), r_{m,equity}\right)}{Var\left(r_{m,equity}\right)}\left[E\left(r_m\right) - rf_{debt}\left(1-t_c\right)\right]}{\left(1-t_c\right)}$$

$$= rf_{debt} + \frac{Cov\left(r_{debt}, r_{m,equity}\right)}{Var\left(r_{m,equity}\right)}\left[E\left(r_m\right) - rf_{debt}\left(1-t_c\right)\right]$$